Equivalence Scales:

An Update and an Extension*

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Abstract

We look at equivalence scales using semiparametric regression in order to update estimates for South Africa. We update the analysis from Yatchew et al. (2003), which is based on data from 1993.

Keywords: Equivalence Scales, Semiparametric, Nonparametric

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1 Introduction

Equivalence scales are used widely in the applied health and development literature. For the most part, the applied literature makes use of either a weighted child economies of scale version, as in (1), or a household size economy of scale version, as in (6).

$$E = (A + \beta K)^{\gamma} \tag{1}$$

$$E = (A + K)^{\delta} \tag{2}$$

Although widely used, much of the applied literature incorporates either (seemingly) arbitrary values for the parameters or values that have been estimated quite a number of years ago, and may not be valid.

In health, in particular, this is acute. Adult equivalence is an important component in the financial risk protection and equity literatures. Many of the assumptions are built upon estimates in Xu et al. (2003). For South Africa, those estimates are taken from the 1993 PSLSD, which was collected nearly one quarter a century ago. Although Yatchew et al. (2003) provide a different set of estimates, they are also based on the same 1993 PSLSD. Recently, Posel et al. (2016) have undertaken a comparison across male and female headed households; however, their analysis did not include estimation. Rather, the analysis was based on simulated hypotheticals. Anonymous (2016) do address these concerns, but consider only linear estimates of the food expenditure share function. In other words, they do not concern themselves with base independence, and do not, thus, consider other shares.

Therefore, we aim to update the equivalence scale estimates that are available in South Africa, making use of relatively recent data, and incorporating concerns over base independence.[1] We semiparametrically estimate versions of both (1) and (6), as well as making simple binary comparisons across households.

[1] We would also like to provide a more nuanced view of the change over time in the country, making use of the data that is available. Furthermore, we would like to include additional datasets, to see how consistent the estimate is. However, that has not been done, yet.

2 Literature Review

Recent literature has noted that poverty measures, which are influenced by the choice of equivalence scale, may not be appropriately esitmated in South Africa (Posel et al., 2016); however, this research did not include directly estimated updated equivalence scales. Instead, they provide a more nuanced analysis. They find that Africans spend more on food than non-Africans, while the food expenditure share to total expenditure gradient is a bit flatter amongst Africans. When combining these features, equivalence scales should generally be lower for Africans, [2] such that the economies of scale (γ) in African households is lower (thus lower additional costs per member), while child costs are relatively higher in African households. They focus on understanding the effect of revised equivalence scales on poverty. On the other hand, citetposeletal2016wp undertake a parametric analysis of equivalence scales, using the demographic version of a standard Working-Leser share, where j represents the age group to be considered.

$$w = \beta_0 + \beta_1 \ln(\frac{x}{n}) + \sum_j \gamma_j n_j + Z\delta. \tag{3}$$

Defining a reference group to have an r superscript, it is possible to use these estimates to calculate an equivalence scale based on the relative difference in expenditure required to reach the same share.

$$E = \frac{n}{n^r} \exp\left(\frac{\sum_j \gamma_j \left(n_j - n_j^r\right)}{\beta_1}\right) \tag{4}$$

Although such an approach does have concerns, many of which are discussed by the authors – using food could be problematic, because children consume mostly food, thus food shares would tend to be higher in a household with children (especially very young children) (see Nicholson, 1976). Relatedly, Deaton (1987), and many others more recently, remind us that household size and structure should be part of the choice set of households, and, therefore, we should always be somewhat circumspect when interpreting these values. Furthermore, as is unfortunately common with South African data, they do not incorporate price data (an exercise that is not easy to do); thus, there is likely heterogeneity: Africans might live in areas with relatively higher prices than

[2] In their analysis, they approximate equation (1) using White and Massett (2002).

non-Africans or vice versa. Finally, they do not consider base independence, although there is no reason to suspect it is any more or less likely to hold in South Africa.

Base independence has featured in the literature at least since Pollak and Wales (1979), although it was not initially discussed in that way. Blundell and Lewbel (1991) take Pollak and Wales's (1979) notion that equivalence scales cannot be identified from demand curves. Even though demands can be uniquely recovered from cost functions, cost functions (or expenditure functions) cannot be uniquely recovered from demand curves. Since demands tell us about preferences related to household structure in consumption space, while equivalence requires information about preferences in household structure - consumption space. Because these spaces are not the same, identification is not generally unique. However, cost of living indexes are estimable, which allows the researcher to back out relative equivalence scales, which are ratios of cost of living indexes for different types of households.

Base independence (Blundell and Lewbel, 1991) is a functional form assumption that forces equivalence scales to be independent of the base level of utility. If the cost function is given by

$$c(p,u,z) = m(p,z)G(p,u) \tag{5}$$

the equivalence scale,

$$E(p,z,z^r) = \frac{c(p,u,z)}{c(p,u,z^r)} = \frac{m(p,z)G(p,u)}{m(p,z^r)G(p,u)} \tag{6}$$

is independent of u. Given the fact that we do not have price data, we are forced to work with Engel "scales"; thus, any results (so far) assume base independence.[3]

Blackorby and Donaldson (1993) offer a slightly different view of base independence. The assumption, outlined in equation (6), is a restriction on both interpersonal and intrapersonal utility comparisons, where the latter is an assumptions on the household social evaluation function, which they they relate to individual utility functions.[4] They,

[3] They make an additional point; essentially, if we can find a way to tie down choices over some things, for example, kids, it might be possible to tie down the equivalence scales. Other researchers, not yet incorportated into this manuscript, have considered including other forms of revealed preference or psychometric information, for example. We have not considered that, yet.

[4] They also rely on Ordinal Full Comparability Plus; to oversimplify, this assumption relies on something already noted in Pollak and Wales (1979) and discussed above when discussing Blundell

instead, define income-ratio comparability to allow for a simple scaling of income to not affect utility "comparisons". Thus, it is not so surprising that this fits within the concept of base independence, and, in fact, is identical. Given their model,they are able to show that equivalence scales can be estimated from the data. They further show that it can not be estimated within the context of the almost ideal demand system, which has Working-Leser share functions. In other words, the estimates in Anonymous (2016) are likely to be problematic.

Empirically, a number of researchers have attempted to incorporate and test base independence. At this point, we only consider a few of those papers. As Pendakur (1999) notes, an accurate equivalence scale may allow policymakers to design transfer programmes that do not create incentives for program participants to change their household type to increase their level of welfare; thus, such an analysis is necessary. He estimates food expenditure shares in subsets of households – by type; these estimates are used to calculate equivalence scales under the assumption of base independence, finding only limited support. He argues that the failure of base independence arises from "child-goods". Under base-independence, firstly, childless couples should have zero expenditure (or close to it, although they might buy gifts) with a zero gradient; therefore, any couples with children should also see 'flat' curves.[5]

For our analysis, we make use of ideas presened by Yatchew et al. (2003), who offer a different approach to estimation than suggested by Pendakur (1999). In particular, they offer a model that has a few simple parameters in it, which are easier to interpret. Furthermore, it allows for estimation across a range of households; wherease Pendakur's (1999) esimates were pairwise by type of household. Intuitively, they offer a minor generalisation of equation (1) to rewrite base independence as in (7), and incorporate it

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and Lewbel (1991), forcing any monotonic transformation of utility to not include a measure of the demographic structure of the household.

[5]Independence of Base (IB) or Equivalence Scale Exactness (ESE) has the consequence that the budget-share equation for any commodity decomposes additively into a function of (u, p) and a function of (p, z^r). The first of these is the share equation for the reference household. From this decomposition, the income elasticity of demand for any pure children's good, such as day-care is one, which is probably not satisfied by real household preferences. See Blackorby and Donaldson (1993), as well, since much of this is directly quoted from there.

into a semiparametric model.

$$y_b = f_b\left(p, x_b\right) = f_a\left(p, \frac{x_b}{\Delta_b(p)}\right) + \eta_b(p) \tag{7}$$

Equation (7) is rewritten as a simple index model that can be estimated via grid search.

$$y = f\left(\ln x - z\delta\right) + z\eta + \varepsilon \tag{8}$$

In the model equation (8), z represents a vector of household types. They estimate this using different functions of $z\delta$, nearest-neighbor methods, and GLS (although I oversimplify). Clearly, this paper can be revisited to add newer data. It can also be extended by estimating the different 'effects' entirely nonparametrically. They suggest Engel's method assumes $\eta = 0$, since it is based only on horizontal shifts. Similarly, Rothbarth's method presumably focuses only on adult good expenditure, or at least non-food expenditure, and also sets $\eta = 0$. In our analysis, we do not set η to zero.[6]

Although we do not consider Donaldson and Pendakur's (2003) generalisation of equivalence scale exactness, we would like to do so in future. In their model, they assume the equivalent scale, itself, has a Cobb-Douglas form, where the parameters are functions of prices and household structure.

$$x^e = X(p, x, z) = \gamma(p, z)x^{\kappa(p,z)} \tag{9}$$

Their results suggest that equivalence scales can depend on overall expenditure, and still be identified: only up to a point, and only because of the functional form specification.

3 Methods

3.1 A Model

This section can probably be placed in an appendix, but I will leave it here for now.

Let us begin with a notion of equal utility for different types of households, where

[6] Doing so would allow us to follow Ichimura, directly, as it is implemented in Hayfield and Racine's (2008) nonparametric package for R (R Core Team, 2017).

the scale might be a function of total expenditure.

$$V\left(p,x,z\right) = V\left(p, \frac{x}{\Delta(p,x,z)}, z^r\right) \tag{10}$$

Recall from basic micro, the relationship between indirect utility and expenditure shares.

$$w_j(p,x,z) = -\frac{\partial V/\partial \ln p_j}{\partial V/\partial \ln x} = -\frac{\partial V/\partial p_j}{\partial V/\partial x} \times \frac{p_j}{x} \tag{11}$$

We apply the semi-log version of Roy's Identity to yield our estimating equations.

For the reference household, the share equation is based on simple substitution of terms, yielding the share.

$$w_j(p,x^r,z^r) = -\frac{\partial V/\partial \ln p_j}{\partial V/\partial \ln x^r} \tag{12}$$

Keep in mind the assumption that $x^r \equiv x/\Delta$; thus, $\Delta(p,x^r,z^r) = 1$.

For any comparator household, we split the numerator and denominator into separate components. First, the numerator, although we include the term following $\times$ in equation (11); we drop the specific share notation j for convenience, such that subscripts represent the term over which the derivative is taken.

$$\begin{aligned} -\frac{\partial V}{\partial p_j} \times \frac{p_j}{x} &= \left[-V_p - V_x \frac{x}{\Delta^2}\left(-\frac{\partial \Delta}{\partial p}\right)\right]\frac{p}{x} \\ &= -V_p \times \frac{p}{x} + \frac{V_x}{\Delta}\left(\frac{\partial \Delta}{\partial p} \times \frac{p}{\Delta}\right) \\ &= -\frac{V_p p}{x} + \frac{V_x}{\Delta}\eta_{\Delta p} \end{aligned} \tag{13}$$

Next, we turn our attention to the denominator.

$$\begin{aligned} \frac{\partial V}{\partial x} &= \frac{V_x}{\Delta} + V_x \frac{x}{\Delta^2}\left(-\frac{\partial \Delta}{\partial x}\right) \\ &= \frac{V_x}{\Delta}\left(1 - \frac{\partial \Delta}{\partial x}\frac{x}{\Delta}\right) \\ &= \frac{V_x}{\Delta}\left(1 - \eta_{\Delta x}\right) \end{aligned} \tag{14}$$

Placing equations (13) and (14) into (11) results in our comparison with the reference share.

$$
\begin{aligned}
w_j(p,x,z) &= -\left(\frac{V_p p}{x} + \frac{V_x}{\Delta}\eta_{\Delta p}\right)\left(\frac{\Delta}{V_x(1-\eta_{\Delta x})}\right) \\
&= \left(-\frac{V_p p}{x}\frac{\Delta}{V_x(1-\eta_{\Delta x})}\right) + \left(\frac{V_x \eta_{\Delta p}}{\Delta}\frac{\Delta}{V_x(1-\eta_{\Delta x})}\right) \\
&= \left[\frac{1}{1-\eta_{\Delta x}}\right]\left(-\frac{V_p}{V_x}\frac{p\Delta}{x}\right) + \frac{\eta_{\Delta p}}{1-\eta_{\Delta x}} \\
&= \frac{1}{1-\eta_{\Delta x}}\left[\left(-\frac{V_p}{V_x}\frac{p}{x/\Delta}\right) + \eta_{\Delta p}\right]
\end{aligned}
\tag{15}
$$

Writing more clearly,

$$
w_j(p,x,z) = \frac{w_j(p,x^r,z^r) + \eta_{\Delta p}}{1-\eta_{\Delta x}} \tag{16}
$$

Under equivalence scale exactness, the scale cannot depend on utility, and, thus, cannot depend on expenditure.

$$
w_j(p,x,z) = w_j(p,x^r,z^r) + \eta_{\Delta p} \tag{17}
$$

Unfortunately, we do not have prices, thus, we are left to estimate

$$
w_j(x,z) = w_j(x^r,z^r) + \eta_p \tag{18}
$$

In effect, this is Yatchew et al.'s (2003) premise, see (7).

3.2 Empirical Methods

The empirical model to be estimated is premised on (8), repeated below as (19) for ease of reference.

$$
y = f\left(\ln x - z\delta\right) + z\eta + \varepsilon \tag{19}
$$

Base independence identification requires nonlinearity, but also vertical and horizontal translations, as implied by (19). It is also possible to replace $z\delta$ with $(A+\beta K)^\gamma$, which would allow for a more standard adult equivalence interpretation.[7]

Initially, however, we split the data into comparable groups, where the dimension of comparison is based either on having one more adult, while keeping the number of children the same, or on having one more child, while keeping the number of adults

[7]Do I want to include Deaton's approach for comparative purposes?

constant.[8] In these initial models, we are, instead, estimating (20)

$$y = f\left(\ln x - \delta\right) + \eta + \varepsilon \tag{20}$$

4 Data

Sorry, this part of the paper is woefully short of appropriate information. Further, I do need to present a useful table of summary information with respect to the data.

4.1 Descriptive Statistics

5 Results

5.1 Semiparametrically Estimated Parameters

5.2 Resultant Equivalence Scales

Table 1: Equivalence Scale Estimates for All Households

		$\Delta = (A+K)^{\theta}$		$\Delta = (A+\beta_1 K)^{\beta_2}$	
Adults	Kids	$\hat{\Delta}$ (s.e.)	$\hat{\delta}$ (s.e.)	$\hat{\Delta}$ (s.e.)	$\hat{\delta}$ (s.e.)
1	0	1.0000	0.0000	1.0000	0.0000
		(0.000)	(0.000)	(0.000)	(0.000)
1	1	1.3449	0.2963	1.1721	0.1588
		(0.022)	(0.016)	(0.032)	(0.032)
1	2	1.5994	0.4697	1.3166	0.2750
		(0.041)	(0.026)	(0.058)	(0.051)
1	3	1.8088	0.5926	1.4431	0.3668
		(0.059)	(0.033)	(0.082)	(0.064)
1	4	1.9898	0.6880	1.5567	0.4426
		(0.076)	(0.038)	(0.103)	(0.075)
1	5	2.1511	0.7660	1.6606	0.5072
		(0.091)	(0.042)	(0.123)	(0.084)
2	0	1.3449	0.2963	1.3543	0.3033
		(0.022)	(0.016)	(0.045)	(0.033)
2	1	1.5994	0.4697	1.4767	0.3898
		(0.041)	(0.026)	(0.072)	(0.051)
2	2	1.8088	0.5926	1.5873	0.4620

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[8] We can extend this by adding race, gender or location effects. Do we want to do that, too?

		$\Delta = (A + K)^{\theta}$		$\Delta = (A + \beta_1 K)^{\beta_2}$	
		$\hat{\Delta}$	$\hat{\delta}$	$\hat{\Delta}$	$\hat{\delta}$
Adults	Kids	(s.e.)	(s.e.)	(s.e.)	(s.e.)
		(0.059)	(0.033)	(0.095)	(0.064)
2	3	1.9898	0.6880	1.6888	0.5240
		(0.076)	(0.038)	(0.117)	(0.075)
2	4	2.1511	0.7660	1.7829	0.5783
		(0.091)	(0.042)	(0.137)	(0.083)
2	5	2.2976	0.8319	1.8711	0.6265
		(0.105)	(0.046)	(0.156)	(0.091)
3	0	1.5994	0.4697	1.6171	0.4806
		(0.041)	(0.026)	(0.085)	(0.053)
3	1	1.8088	0.5926	1.7164	0.5402
		(0.059)	(0.033)	(0.109)	(0.065)
3	2	1.9898	0.6880	1.8087	0.5926
		(0.076)	(0.038)	(0.130)	(0.075)
3	3	2.1511	0.7660	1.8954	0.6394
		(0.091)	(0.042)	(0.151)	(0.084)
3	4	2.2976	0.8319	1.9772	0.6817
		(0.105)	(0.046)	(0.169)	(0.091)
3	5	2.4326	0.8890	2.0549	0.7202
		(0.119)	(0.049)	(0.188)	(0.097)
4	0	1.8088	0.5926	1.8340	0.6065
		(0.059)	(0.033)	(0.122)	(0.067)
4	1	1.9898	0.6880	1.9192	0.6519
		(0.076)	(0.038)	(0.144)	(0.076)
4	2	2.1511	0.7660	1.9998	0.6931
		(0.091)	(0.042)	(0.164)	(0.084)
4	3	2.2976	0.8319	2.0764	0.7307
		(0.105)	(0.046)	(0.182)	(0.091)
4	4	2.4326	0.8890	2.1496	0.7653
		(0.119)	(0.049)	(0.200)	(0.097)
4	5	2.5582	0.9393	2.2197	0.7974
		(0.133)	(0.052)	(0.218)	(0.103)
5	0	1.9898	0.6880	2.0221	0.7041
		(0.076)	(0.038)	(0.156)	(0.077)
5	1	2.1511	0.7660	2.0977	0.7408
		(0.091)	(0.042)	(0.176)	(0.085)
5	2	2.2976	0.8319	2.1699	0.7747
		(0.105)	(0.046)	(0.195)	(0.092)
5	3	2.4326	0.8890	2.2392	0.8061
		(0.119)	(0.049)	(0.213)	(0.098)
5	4	2.5582	0.9393	2.3058	0.8354
		(0.133)	(0.052)	(0.230)	(0.103)
5	5	2.6761	0.9844	2.3700	0.8629
		(0.145)	(0.054)	(0.246)	(0.108)

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Adults	Kids	$\Delta = (A+K)^{\theta}$ $\hat{\Delta}$ (s.e.)	$\hat{\delta}$ (s.e.)	$\Delta = (A+\beta_1 K)^{\beta_2}$ $\hat{\Delta}$ (s.e.)	$\hat{\delta}$ (s.e.)
6	0	2.1511	0.7660	2.1900	0.7839
		(0.091)	(0.042)	(0.189)	(0.086)
6	1	2.2976	0.8319	2.2585	0.8147
		(0.105)	(0.046)	(0.207)	(0.093)
6	2	2.4326	0.8890	2.3244	0.8435
		(0.119)	(0.049)	(0.225)	(0.098)
6	3	2.5582	0.9393	2.3880	0.8704
		(0.133)	(0.052)	(0.242)	(0.104)
6	4	2.6761	0.9844	2.4495	0.8959
		(0.145)	(0.054)	(0.258)	(0.108)
6	5	2.7874	1.0251	2.5090	0.9199
		(0.158)	(0.057)	(0.274)	(0.113)
7	0	2.2976	0.8319	2.3428	0.8513
		(0.105)	(0.046)	(0.219)	(0.093)
7	1	2.4326	0.8890	2.4057	0.8779
		(0.119)	(0.049)	(0.237)	(0.099)
7	2	2.5582	0.9393	2.4667	0.9029
		(0.133)	(0.052)	(0.254)	(0.104)
7	3	2.6761	0.9844	2.5257	0.9265
		(0.145)	(0.054)	(0.270)	(0.109)
7	4	2.7874	1.0251	2.5830	0.9490
		(0.158)	(0.057)	(0.285)	(0.113)
7	5	2.8930	1.0623	2.6388	0.9703
		(0.170)	(0.059)	(0.301)	(0.117)

Equivalence scale estimates from equation – reference – and equation – reference – for All households.

Table 2: Equivalence Scale Estimates for Black Households

Adults	Kids	$\Delta = (A+K)^{\theta}$ $\hat{\Delta}$ (s.e.)	$\hat{\delta}$ (s.e.)	$\Delta = (A+\beta_1 K)^{\beta_2}$ $\hat{\Delta}$ (s.e.)	$\hat{\delta}$ (s.e.)
1	0	1.0000	0.0000	1.0000	0.0000
		(0.000)	(0.000)	(0.000)	(0.000)
1	1	1.3036	0.2651	1.1388	0.1300
		(0.024)	(0.018)	(0.033)	(0.034)
1	2	1.5223	0.4202	1.2554	0.2274
		(0.044)	(0.029)	(0.060)	(0.054)
1	3	1.6994	0.5303	1.3572	0.3054

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		$\Delta = (A+K)^{\theta}$		$\Delta = (A+\beta_1 K)^{\beta_2}$	
		$\hat{\Delta}$	$\hat{\delta}$	$\hat{\Delta}$	$\hat{\delta}$
Adults	Kids	(s.e.)	(s.e.)	(s.e.)	(s.e.)
		(0.062)	(0.036)	(0.082)	(0.069)
1	4	1.8508	0.6156	1.4484	0.3705
		(0.078)	(0.042)	(0.103)	(0.080)
1	5	1.9845	0.6853	1.5315	0.4263
		(0.093)	(0.047)	(0.122)	(0.089)
2	0	1.3036	0.2651	1.3127	0.2721
		(0.024)	(0.018)	(0.046)	(0.035)
2	1	1.5223	0.4202	1.4083	0.3424
		(0.044)	(0.029)	(0.073)	(0.054)
2	2	1.6994	0.5303	1.4948	0.4020
		(0.062)	(0.036)	(0.096)	(0.068)
2	3	1.8508	0.6156	1.5742	0.4538
		(0.078)	(0.042)	(0.116)	(0.079)
2	4	1.9845	0.6853	1.6479	0.4995
		(0.093)	(0.047)	(0.135)	(0.088)
2	5	2.1050	0.7443	1.7167	0.5404
		(0.107)	(0.051)	(0.153)	(0.096)
3	0	1.5223	0.4202	1.5391	0.4312
		(0.044)	(0.029)	(0.086)	(0.056)
3	1	1.6994	0.5303	1.6152	0.4795
		(0.062)	(0.036)	(0.109)	(0.069)
3	2	1.8508	0.6156	1.6861	0.5224
		(0.078)	(0.042)	(0.129)	(0.080)
3	3	1.9845	0.6853	1.7527	0.5612
		(0.093)	(0.047)	(0.148)	(0.089)
3	4	2.1050	0.7443	1.8156	0.5964
		(0.107)	(0.051)	(0.166)	(0.096)
3	5	2.2153	0.7954	1.8753	0.6287
		(0.120)	(0.054)	(0.182)	(0.103)
4	0	1.6994	0.5303	1.7231	0.5441
		(0.062)	(0.036)	(0.122)	(0.071)
4	1	1.8508	0.6156	1.7876	0.5809
		(0.078)	(0.042)	(0.142)	(0.081)
4	2	1.9845	0.6853	1.8486	0.6145
		(0.093)	(0.047)	(0.161)	(0.089)
4	3	2.1050	0.7443	1.9067	0.6454
		(0.107)	(0.051)	(0.178)	(0.097)
4	4	2.2153	0.7954	1.9622	0.6741
		(0.120)	(0.054)	(0.195)	(0.103)
4	5	2.3174	0.8404	2.0154	0.7008
		(0.133)	(0.057)	(0.210)	(0.109)
5	0	1.8508	0.6156	1.8808	0.6317
		(0.078)	(0.042)	(0.154)	(0.082)

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		$\Delta = (A+K)^{\theta}$		$\Delta = (A+\beta_1 K)^{\beta_2}$	
		$\hat{\Delta}$	$\hat{\delta}$	$\hat{\Delta}$	$\hat{\delta}$
Adults	Kids	(s.e.)	(s.e.)	(s.e.)	(s.e.)
5	1	1.9845	0.6853	1.9374	0.6614
		(0.093)	(0.047)	(0.173)	(0.090)
5	2	2.1050	0.7443	1.9916	0.6889
		(0.107)	(0.051)	(0.190)	(0.097)
5	3	2.2153	0.7954	2.0436	0.7147
		(0.120)	(0.054)	(0.206)	(0.104)
5	4	2.3174	0.8404	2.0936	0.7389
		(0.133)	(0.057)	(0.222)	(0.109)
5	5	2.4127	0.8807	2.1418	0.7617
		(0.145)	(0.060)	(0.237)	(0.115)
6	0	1.9845	0.6853	2.0203	0.7033
		(0.093)	(0.047)	(0.184)	(0.091)
6	1	2.1050	0.7443	2.0712	0.7281
		(0.107)	(0.051)	(0.202)	(0.098)
6	2	2.2153	0.7954	2.1202	0.7515
		(0.120)	(0.054)	(0.218)	(0.104)
6	3	2.3174	0.8404	2.1676	0.7736
		(0.133)	(0.057)	(0.233)	(0.110)
6	4	2.4127	0.8807	2.2133	0.7945
		(0.145)	(0.060)	(0.248)	(0.115)
6	5	2.5023	0.9172	2.2577	0.8143
		(0.157)	(0.063)	(0.262)	(0.120)
7	0	2.1050	0.7443	2.1464	0.7638
		(0.107)	(0.051)	(0.213)	(0.099)
7	1	2.2153	0.7954	2.1928	0.7852
		(0.120)	(0.054)	(0.229)	(0.105)
7	2	2.3174	0.8404	2.2378	0.8055
		(0.133)	(0.057)	(0.244)	(0.111)
7	3	2.4127	0.8807	2.2814	0.8248
		(0.145)	(0.060)	(0.259)	(0.116)
7	4	2.5023	0.9172	2.3238	0.8432
		(0.157)	(0.063)	(0.273)	(0.120)
7	5	2.5869	0.9505	2.3650	0.8608
		(0.168)	(0.065)	(0.286)	(0.124)

Equivalence scale estimates from equation – reference – and equation – reference – for Black households.

Table 3: Equivalence Scale Estimates for Coloured Households

		$\Delta = (A + K)^{\theta}$		$\Delta = (A + \beta_1 K)^{\beta_2}$	
		$\hat{\Delta}$	$\hat{\delta}$	$\hat{\Delta}$	$\hat{\delta}$
Adults	Kids	(s.e.)	(s.e.)	(s.e.)	(s.e.)
1	0	1.0000	0.0000	1.0000	0.0000
		(0.000)	(0.000)	(0.000)	(0.000)
1	1	1.2991	0.2617	1.1405	0.1315
		(0.093)	(0.072)	(0.124)	(0.120)
1	2	1.5140	0.4147	1.2586	0.2300
		(0.172)	(0.113)	(0.226)	(0.196)
1	3	1.6876	0.5233	1.3617	0.3087
		(0.241)	(0.143)	(0.315)	(0.251)
2	0	1.2991	0.2617	1.3149	0.2738
		(0.093)	(0.072)	(0.188)	(0.143)
2	1	1.5140	0.4147	1.4120	0.3450
		(0.172)	(0.113)	(0.286)	(0.209)
2	2	1.6876	0.5233	1.4997	0.4053
		(0.241)	(0.143)	(0.374)	(0.260)
2	3	1.8360	0.6076	1.5803	0.4576
		(0.305)	(0.166)	(0.454)	(0.301)
3	0	1.5140	0.4147	1.5433	0.4340
		(0.172)	(0.113)	(0.350)	(0.227)
3	1	1.6876	0.5233	1.6206	0.4828
		(0.241)	(0.143)	(0.435)	(0.273)
3	2	1.8360	0.6076	1.6926	0.5263
		(0.305)	(0.166)	(0.513)	(0.311)
3	3	1.9668	0.6764	1.7602	0.5654
		(0.364)	(0.185)	(0.585)	(0.343)
4	0	1.6876	0.5233	1.7291	0.5476
		(0.241)	(0.143)	(0.494)	(0.286)
4	1	1.8360	0.6076	1.7946	0.5848
		(0.305)	(0.166)	(0.570)	(0.321)
4	2	1.9668	0.6764	1.8567	0.6188
		(0.364)	(0.185)	(0.641)	(0.352)
4	3	2.0846	0.7346	1.9157	0.6501
		(0.418)	(0.201)	(0.708)	(0.378)
5	0	1.8360	0.6076	1.8884	0.6357
		(0.305)	(0.166)	(0.627)	(0.332)
5	1	1.9668	0.6764	1.9460	0.6658
		(0.364)	(0.185)	(0.696)	(0.361)
5	2	2.0846	0.7346	2.0011	0.6937
		(0.418)	(0.201)	(0.762)	(0.386)
5	3	2.1924	0.7850	2.0539	0.7197
		(0.470)	(0.215)	(0.825)	(0.409)

Equivalence scale estimates from equation – reference – and equation – reference – for Coloured households.

Table 4: Equivalence Scale Estimates for White Households

		$\Delta = (A+K)^{\theta}$		$\Delta = (A+\beta_1 K)^{\beta_2}$	
		$\hat{\Delta}$	$\hat{\delta}$	$\hat{\Delta}$	$\hat{\delta}$
Adults	Kids	(s.e.)	(s.e.)	(s.e.)	(s.e.)
1	0	1.0000	0.0000	1.0000	0.0000
		(0.000)	(0.000)	(0.000)	(0.000)
1	1	1.4020	0.3379	1.2028	0.1846
		(0.211)	(0.150)	(0.384)	(0.360)
1	2	1.7084	0.5356	1.3733	0.3172
		(0.407)	(0.238)	(0.718)	(0.583)
2	0	1.4020	0.3379	1.3899	0.3292
		(0.211)	(0.150)	(0.569)	(0.409)
2	1	1.7084	0.5356	1.5380	0.4305
		(0.407)	(0.238)	(0.906)	(0.612)
2	2	1.9656	0.6758	1.6717	0.5139
		(0.591)	(0.301)	(1.213)	(0.764)
3	0	1.7084	0.5356	1.6851	0.5218
		(0.407)	(0.238)	(1.093)	(0.649)
3	1	1.9656	0.6758	1.8070	0.5917
		(0.591)	(0.301)	(1.400)	(0.791)
3	2	2.1915	0.7846	1.9204	0.6525
		(0.765)	(0.349)	(1.685)	(0.906)
4	0	1.9656	0.6758	1.9319	0.6585
		(0.591)	(0.301)	(1.582)	(0.819)
4	1	2.1915	0.7846	2.0376	0.7118
		(0.765)	(0.349)	(1.866)	(0.928)
4	2	2.3952	0.8735	2.1377	0.7597
		(0.930)	(0.388)	(2.134)	(1.021)

Equivalence scale estimates from equation – reference – and equation – reference – for white households.

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Table A.1: Parameter Estimates from Semiparametric Models

	$\hat{\theta}$ (s.e.)	$\hat{\eta}$ (s.e.)	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_2$ (s.e.)	$\hat{\eta}_1$ (s.e.)	$\hat{\eta}_2$ (s.e.)
All HH SP	0.4275	0.0040	1.0000	0.4375	0.0000	0.0081
N= 24206	(0.024)	(0.001)	(0.031)	(0.048)	(0.001)	(0.001)
Black HH SP	0.3825	0.0047	1.0000	0.3925	-0.0004	0.0098
N= 19143	(0.026)	(0.001)	(0.035)	(0.051)	(0.002)	(0.001)
Colour HH SP	0.3775	0.0043	0.8150	0.3950	0.0038	0.0050
N= 2442	(0.103)	(0.004)	(0.126)	(0.206)	(0.006)	(0.006)
White HH SP	0.4875	-0.0005	0.9000	0.4750	0.0017	-0.0023
N= 1865	(0.217)	(0.004)	(0.270)	(0.591)	(0.006)	(0.007)

Parameter estimates from equation – reference – and equation – reference – for all households.

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A Parameter Estimates