Balance Sheet Policies and Financial Stability

by

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Abstract

Still needs to be written.

1 Introduction

The policy arsenal of central banks has undergone a noticeable expansion in recent years. Policymakers are no longer limited to the one monetary policy instrument, a short-term interest rate, which was central to the New Neoclassical synthesis (Woodford, 2003). Balance sheets of monetary authorities are playing an increasingly important role in policy formulation, as recently emphasised by Bernanke (2011). Policies that increase the size and composition of central bank balance sheets are now used in conjunction with interest rate policy to achieve, simultaneously, price- and financial stability[1].

Central banks have traditionally, since the 1970s, considered the balance sheet only in its capacity to steer the overnight rate towards the policy target. The objective of this paper is to determine in what capacity central bank lending can be used to support financial stability. My hypothesis is most similar to that of Goodhart et al. (2011), who consider the potential role of the central bank's balance sheet in the pursuit of financial stability, but I differ on a few specifics.

First, my model is set in a dynamic general equilibrium setting, which - unlike a comparative static setting - allows the researcher to consider the dynamics of the economy. The model presented here is similar to the approach adopted in de Walque et al. (2010). However, I embedded the essential properties of their real business cycle (RBC) model into a New-Keynesian setting with price and wage rigidities, which allows for a richer understanding of the implications of monetary policy.

Second, I endeavoured to provide a more realistic presentation of central bank lending. In Goodhart et al. (2011), the monetary base is mapped one-to-one onto the interest rate, which is not an accurate representation of modern central bank practice. Two important contributions to the literature that also look at the impact of monetary injections on financial stability are de Walque et al. (2010) and Dib (2010a). However, in both of these papers, the monetary authority is structured with direct money injections from the central bank to the commercial (merchant/lending) bank. My paper differs in that it includes collateralised repurchase agreements, as first modelled in Reynard and Schabert (2009) and more recently in Schabert (2015) and Hörmann and Schabert (2015).

Third, the baseline model is extended to include long-term securities, allowing the central bank to affect the economy through changes in the composition of its balance sheet. Balance sheet policy of this type usually attempts to influence the price of a specific asset class. However, there

[1]Macroprudential policy measures, as later discussed, are also being used to varying degrees by central banks, existing on occasion as part of the monetary policy space and at other times separated by independence from monetary authorities.

are other avenues through which asset purchases in my model might impact the macroeconomy, specifically in the financial sector. Changes to the baseline model primarily affect three sections of the model, namely the merchant bank, central bank and government.

The article is structured as follows. First, I provide an overview of the related literature, ranging from issues pertaining to financial frictions in DSGE models to methods for employing central bank lending in mainstream models. Second, the model is presented, with the integration of the mechanism proposed by Schabert (2015) in the banking and public sector. The model is kept as simple as possible, abstracting from financial frictions in the demand side of the credit market. Third, changes in the composition of the central bank balance sheet are introduced, to complement the structure of the baseline model. Fourth, a discussion follows on the results of the impulse response functions following (i) a balance sheet expansion and (ii) an increase in the policy rate (monetary tightening) (iii) change in the composition of assets held on the central bank balance sheet. Finally, the last section concludes.

2 Literature Review

The two decades preceding the Great Recession were marked by an unprecedented consensus on the "intellectual and institutional framework for monetary policy" (Bernanke, 2011). At the heart of the consensus is the dynamic general equilibrium framework, which was pioneered by Leeper and Sims (1994) and Schorfheide (2000). It was then further propelled into the mainstream by the seminal contributions of, among others, Woodford (2003), Smets and Wouters (2003, 2007) and Christiano et al. (2005). This macroeconometric modelling paradigm has been implemented universally by central banks, who were able to utilise the models successfully to understand the consequences of policy actions better, and thereby achieve macroeconomic stability. However, these models were not equipped to forestall financial market failure.

Economics, as a discipline, has developed with the use of theoretical frameworks that often ignore knotty real-world frictions in order to remain tractable and computationally feasible (de Walque et al., 2010). Theorists make assumptions that reduce complex real-world interactions into digestible mathematical equations. Some of these assumptions have been questioned in the wake of the international financial crisis. A particular concern is the idea that financial markets are perfect and complete, with the implication that financial shocks are irrelevant to real economic outcomes (Roger and Vlcek, 2012). There was a commonly held belief that finance, in the first approximation, was irrelevant to business cycle movements (Woodford, 2003). However, owing to recent events, it has been acknowledged that the financial crisis originated from a collapse of financial intermediation, which has cemented the idea that credit market frictions have real implications (Ahn and Tsomocos, 2013). The shortcomings of macroeconomic models revealed during the crisis have led many, such as Kirman (2010), Caballero (2010), Stiglitz (2011), Krugman (2011), DeLong (2011) and Kay (2012), to question the underlying assumptions of DSGE models. The next section discusses several approaches to introducing endogenous financial frictions into the existing New-Keynesian framework.

2.1 Financial Frictions in DSGE Models

Prudent monetary policy, as defined in pre-crisis policy models, is primarily understood with respect to price stability. Financial stability is often considered as a by-product of inflation targeting[2], with financial sectors curiously absent from the majority of mainstream models (Borio, 2014). However, it is now recognised that the "achievement of price stability [...] does not guarantee financial stability" (Goodhart, 2011). The dearth of financial markets in core models is contrasted by a rich vein of research in the periphery that accentuates the role of financial market conditions in propagating cyclical fluctuations. Fisher (1933) and Keynes (1936) were among the earliest to develop an alternative narrative that explained how impaired credit markets[3] could substantially contribute to a decline in the real economy[4]. More recently, in the post-war period, these points have been brought to the fore by the contributions of Minsky (1957, 1982) and Kindleberger (2000). Their argument is directly at odds with the assumption of perfectly competitive financial markets, as used by Modigliani and Miller (1958) in their capital structure irrelevance proposition[5].

2.1.1 Demand-Side Frictions

In perfectly competitive financial markets there are no frictions that limit access to credit, which allows no insight into scenarios where agents are credit constrained. Financial frictions have been introduced into DSGE models to address this limitation. Pioneering contributions to the literature, to include information asymmetries and non-convex transaction costs, were put forward by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Both of these veins of research proposed alterations to the consensus approach, introducing frictions in the demand side of the credit market, where banks act exclusively as intermediaries between households and

[2]This issue is discussed in more detail in Chapter 3.

[3]Examples of impaired credit conditions include, "sharp increases in insolvencies and bankruptcies, rising real debt burdens, collapsing credit prices, and bank failures" (Bernanke et al., 1999).

[4]Theories were developed owing to the events surrounding the Great Depression.

[5]This assumption asserts that the capital structure of banks is largely irrelevant and indeterminate for lending decisions, and thereby, real economic outcomes.

firms. Financial market conditions in these models serve to frame a more complete narrative of the forces that propagate economic growth.

2.1.1.1 External Finance Premium Bernanke and Gertler (1989) were the first to consolidate successfully the financial accelerator mechanism and general equilibrium framework. The inclusion of a financial accelerator is significant as it allows endogenous credit market developments to act both as a source of business cycle fluctuations and as an amplification device. Bernanke and Gertler (1989) postulate that, in the light of tightened financial market conditions, temporary credit shocks could have a strong and persistent effect on the business cycle. To sufficiently constrain credit markets, they integrate the costly state verification framework proposed by Townsend (1979) into a general equilibrium environment.

Costly state verification entails the existence of information asymmetries that obscure the borrower-lender relationship. In the model developed by Bernanke and Gertler (1989), a newly proposed agent, namely the entrepreneur, plays a central role. One of the noteworthy functions of this entrepreneur is its ability to produce capital from consumption goods. In addition, entrepreneurs invest out of their own wealth, as well as taking loans from households. The entrepreneur's net worth is subject to an idiosyncratic shock, where the outcome of the shock is directly observed by the entrepreneur but not the originator of the loan. Lenders would ideally want to know whether entrepreneurs will be able to repay their debt. However, lenders are forced to pay a monitoring cost if they wish to gain information as to the solvency of the entrepreneur. Therefore, borrowing is limited, because monitoring a loan applicant is costly (Brzoza-Brzezina et al., 2011). Efficiency in the process of matching potential borrowers and lenders is reduced (Bernanke et al., 1999).

In this framework, borrowers face a risk premium that decreases with their net worth (Roger and Vlcek, 2012). Standard debt contracts include a premium on the interest rate to cover the cost of default in case of negative wealth shocks (Christiano et al., 2010). An endogenous wedge between the lending and risk free rates is created and is called the external finance premium (Brzoza-Brzezina et al., 2011). In other words, the price of loans is directly affected in this economy. A decrease in price negatively affects the net worth of the entrepreneur and increases the financial friction. The result is lower levels of investment in the next period, coupled with a lower net worth. This feedback mechanism results in strong persistence as the result of tight financial market conditions.

This model setup was further improved by Carlstrom and Fuerst (1997), who incorporated the dynamics into a New-Keynesian DSGE model. Bernanke et al. (1999) added nonlinear capital adjustment costs, to become the workhorse financial accelerator model that is used in many central banks around the world[6]. Important contributions that built on this structure were made by Christiano et al. (2003, 2008) and De Fiore and Uhlig (2005). More recently, Christiano et al. (2014) contributed to the existing paradigm by introducing "idiosyncratic uncertainty in the allocation of capital". Entrepreneurs face uncertainty in the process of converting capital into effective capital, where the magnitude of this uncertainty is modelled as 'risk'.

2.1.1.2 Collateral Constraint The financial accelerator proposed by Kiyotaki and Moore (1997) operates in terms of a different friction than the external finance premium. In this model, the agents differ regarding their time preference. As dictated by their preference, agents are identified as either a lender or borrower. Intermediation exists between these two specific groups. Borrowers differ in this market and are required, by the financial intermediary, to provide collateral for loans. Whereas the friction in Bernanke and Gertler (1989) is based in asymmetric information and affects the price of loans, this friction functions on the basis of incomplete contracts and directly impacts on the specific quantity of loans (Kiyotaki and Moore, 1997).

Owing to criticism by Kocherlakota (2000) regarding the ability of credit constraint frameworks to generate an empirically valid amplification of shocks, several signficant attempts were made to develop a more realistic setting. Cooley et al. (2004) stand out as one of the early attempts at providing a more quantitatively accurate representation. This was achieved by not focusing exclusively on collateralised debt as the primary form of financing for the firm, but rather by including state-contingent financial contracts. Iacoviello (2005) combined elements of the financial accelerator model developed by Bernanke et al. (1999) and the collateral constraint, as in Kiyotaki and Moore (1997). The original contribution of this model is that firms need to provide real estate as collateral, which he motivates both on practical and substantive grounds.

Financial frictions were initially introduced almost exclusively on the demand side of credit markets, with an explicit focus on the balance sheets of non-financial borrowers (Meh and Moran, 2010). Unfortunately, these models neglect the role of financial intermediaries, treating them as a veil (Gertler and Karadi, 2011). Recent events surrounding the financial crisis highlighted the importance of financial shocks originating in the banking sector as a source of business cycle fluctuations (Dib, 2010a). This has resulted in a concerted effort to develop models that explore disruptions in the supply of credit in financial markets (Falagiarda and Saia, 2013). In a liquidity crisis, financial intermediaries become credit constrained and this friction reveals how shocks in the financial economy could have implications for the real economy (de Walque et al., 2010).

[6]Nonlinear capital adjustment costs added another amplification effect to the model.

2.1.2 Supply-Side Frictions

The international financial crisis has highlighted the central role of financial shocks in driving macroeconomic indicators in the real economy (Quadrini, 2011). Researchers are now tasked with the advancement of models that better incorporate frictions on the supply side of credit markets, specifically fostering models with persuasive, well-founded, banking sectors. Investigation has revealed several components that are believed to be crucial in capturing the essence of a sophisticated banking sector. Some of the particular characteristics that need to be incorporated are related to bank capital, interest rate spreads, interbank markets and the possibility of default. I now briefly touch on some of these issues[7].

2.1.2.1 Banking Sectors and Bank Capital Several authors have endeavoured to construct realistic banking sectors. The first wave of models had the specific goal of approximating the bank capital channel. Previously, it was thought that, in line with the assumptions found in Modigliani and Miller (1958), the structure of bank capital was unimportant in lending decisions. However, empirical evidence suggests that the capital position of a bank directly affects bank lending and thereby real economic activity (Roger and Vlcek, 2012). There has been a large influx of these types of models, with many central banks adopting them as the new workhorse. The reason for this is that policymakers wish to incorporate the recent changes to Basel III regulatory requirements. Generally, these models still follow either the collateral constraint or financial accelerator framework[8], with the addition of a bank capital/equity channel.

Early incarnations of this type of model that try to develop the bank capital channel are those of Markovic (2006), Van den Heuvel (2008) and Angeloni and Faia (2009). The work of Markovic (2006)[9] is particularly influential. The most important contribution in his paper, which builds on the financial accelerator framework, is the introduction of a banking sector where banks face adjustment costs in capital accumulation. Asymmetric information between a bank and its shareholders is the source of the adjustment cost, as shareholders need to incur search costs before investing, creating an environment where the continued procurement of bank capital is considered costly (Markovic, 2006).

2.1.2.2 Interest Rate Spreads While the focus on bank capital is crucial, one has to consider that the financial crisis was characterised by widening credit spreads and disruptions in equity markets. Adrian and Shin (2011) point out that financial shocks were transmitted to the real

[7]These themes do not provide a comprehensive account of all the important issues related to developing a functioning banking sector. Some papers will also belong to more than one specific theme.

[8]Sometimes even a hybrid of the two.

[9]A model developed at the Bank of England.

sector, primarily through these channels. Remarkably, in the case of a perfectly competitive banking sector, only one interest rate is considered significant, namely the policy rate (Gerali et al., 2010). The importance of including a time-varying interest rate spread is highlighted in the work of Cúrdia and Woodford (2009, 2010). They include an ad hoc friction in financial intermediation that gives rise to a spread between the loan and policy rate. Increases in the credit spreads are indicative of constrained credit markets, referred to as 'tighter' financial conditions by Cúrdia and Woodford (2009). This credit friction challenges the unrealistic assumption that a single interest rate governs the behaviour of all agents.

Gerali et al. (2010) also indicate a role for the rate at which different interest rates adjust. Their model is built on the back of contributions by Bernanke et al. (1999), Smets and Wouters (2003) and Iacoviello (2005). In this model, financial intermediaries are permitted to set the interest rate charged for deposits collected from households. In this imperfectly competitive banking sector, one observes a wedge between loan rates and the interbank (policy) rate set by the central bank. These authors find that including sticky bank rates produces a financial decelerator (attenuator) effect[10].

In the work of Gertler and Karadi (2011), households are randomly assigned roles as workers or bankers. Bankers provide credit to firms, but constraints are imposed as to the resources they can obtain from deposits and the interbank market. Binding constraints induce a spread between deposit and loan rates. Christiano et al. (2014) present a similar framework, with a spread generated by the possibility of firm failure.

Empirically, one observes time-varying credit spreads, with notable increases in spreads during times of financial turmoil. Cúrdia and Woodford (2010) hypothesise that spreads need to be taken into account by the monetary authority when making policy decisions, specifically through adjustment of the Taylor Rule to incorporate these spreads. Modifying the Taylor Rule in this fashion affords central bankers the opportunity to respond better to shocks originating in the financial sector. Charles Goodhart, in his commentary on the work of Cúrdia and Woodford (2009), welcomes the introduction of the modelled disruption in financial intermediation but provides a scathing critique on the absence of default risk.

2.1.2.3 Interbank Markets and Default The failure of interbank markets is increasingly viewed as central to the damage caused by the financial crisis. In order to model this risk, researchers have to develop an active banking sector where banks are allowed to interact and possibly default. Relatively few papers have incorporated banking sectors and default (Roger and Vlcek, 2012). Perhaps the first to include an explicit banking sector is that of Gerali et al.

[10] In addition to the financial accelerator and bank capital channel identified in their model.

(2010). Their model includes a constraint on bank balance sheets, which affects bank capital, profit and thereby the supply of loans in the economy. Unfortunately, there is no interaction among wholesale banks in this model.

Gertler and Kiyotaki (2010) take another approach to the interbank market. Financial institutions in this setup are exposed to liquidity shocks that distinguish them from one another. These idiosyncratic shocks can potentially disrupt the intermediation process and thereby real economic activity. The biggest shortcoming of this model is that financial institutions resemble a homogenous intermediary in aggregate. In other words, the banking sector does not consist of heterogenous agents, and therefore, can not truly represent an interbank market. However, the work of Gertler et al. (2016) builds on their earlier model, introducing a wholesale banking sector, alongside the retail banking sector. The purpose of this work was to include components of the failure of the shadow banking system during the recent crisis by allowing for runs on the wholesale banks. Several papers, such as Robatto (2014), Gertler and Kiyotaki (2015) and Ferrante (2015), have started to develop the possibility of bank runs in dynamic models.

A relatively new stream of research, which was built on the model developed by Goodhart et al. (2006)[11], provides advances in the development of interbank markets and default. One of the most important contributions of their body of work has been the inclusion of a heterogenous and endogenous banking sector. They abstract from the representative agent approach in modelling the banking system. This allows for the interaction of banks on the interbank market and thereby the possibility of modelling the reactions of commercial banks to certain shocks. Failure of banks, or endogenous default as developed by Shubik and Wilson (1977) and Dubey et al. (2005), is one of the primary features of this model. Failure is a function of the risk preference of banks in this system, with the riskiest banks assigned the highest probability of default. These failures are not isolated events in this model and have system-wide implications for the survival of other banks.

Several authors have adopted this framework to explore various macroeconomic issues. Some of the important articles in this tradition are those of Goodhart et al. (2009), de Walque et al. (2010), Dib (2010a,b), Hilberg and Hollmayr (2011), Martinez and Tsomocos (2011), Carrera and Vega (2012) and Ahn and Tsomocos (2013). My model most closely resembles the work of de Walque et al. (2010).

[11]This article is the culmination of years of research and several other articles are linked in the model's construction (see, for example, Aspachs et al. (2006a); Tsomocos and Zicchino (2005); Aspachs et al. (2006b); Goodhart and Tsomocos (2006)).

2.1.2.4 Macroprudential Policy In this chapter, financial stability is defined in terms of endogenous default, but several other definitions exist[12]. After the crisis, a large body of literature developed around the idea of financial instability and how to combat it. The avenue explored most frequently to target financial stability is that of macroprudential regulation. Approaches that capture the interaction of balance sheet policies and financial stability are more limited.

Galati and Moessner (2013) provide an excellent discussion on macroprudential policies, while the articles by Friedrich et al. (2015) and Collard et al. (2015) discuss the introduction of these policy measures into DSGE models[13]. Of particular interest in this thesis is an article by Woodford (2016) that looks at both the role of quantitative easing and macroprudential policy in combating financial instability. He found that among the three policy instruments[14] available to the central bank, quantitative easing generates the lowest risk for financial instability for a given increase in aggregate demand (Woodford, 2016). In fact, quantitative easing can be used in conjunction with macroprudential policy to almost entirely negate the build-up of financial imbalances.

2.2 Central Bank Lending as Prudential Policy Tool

This section explores the effect of a change in the size of the balance sheet on financial stability[15], through central bank lending. Goodhart et al. (2011) ask a similar question by incorporating the monetary base into their financial fragility framework and analysing its potential for use as a prudential policy tool[16]. It differs from the model presented in this paper in a few significant ways, as depicted in the next section. Their paper can be seen as an intellectual successor to William Poole's (1970) solution of the instrument problem, in that they explore the effectiveness of both the monetary base and interest rates as tools of monetary policy, in a setting where the liquidity effect functions perfectly.

Several papers have considered the optimal instrument choice in achieving price stability. The work of Poole (1970) signified a watershed in the discussion on the instrument problem, with the interest rate winning out as the most effective in achieving price stability. Poole (1970) argued that the policy rate encompasses all the desired characteristics, especially in its tightness

[12]In fact, there is little consensus in the literature on the exact definition of financial stability, see the article by Borio and Drehmann (2009) for a discussion.

[13]While the literature is interesting in its own right, a full literature review is not attempted here.

[14]Short-term policy rate, macroprudential policy and quantitative easing.

[15]There are several definitions of financial stability in the literature. I follow the approach of Goodhart et al. (2006) in this paper.

[16]The paper by Grauwe and Gros (2009) poses a question is in the same vein. They endeavoured to determine whether there is a trade-off between price and financial stability in the use of monetary policy tools.

to inflation[17]. Sargent and Wallace (1975) add forward-looking features in their model and find, in contrast to Poole (1970), that money growth policies have an advantage over interest rate policies. This result has been contested by several authors, namely, McCallum (1981), Woodford (2003) and, more recently Atkeson et al. (2007) and Woodford (2008). However, I am not interested in the instrument problem as it pertains to price stability.

In fact, the question posed here is much simpler. The objective is to design a model that formalizes the idea from the post-crisis discourse that balance sheets matter[18] for financial stability (du Plessis, 2012). Monetary policy research in the last few years has been focused on the central bank's balance sheet, because the short-term interest rate as a conventional tool of monetary policy is limited in its scope to address financial stability. The policy rate is often regarded as a blunt tool against the build-up of financial imbalances accompanying movements in asset prices and credit aggregates (Bernanke, 2011)[19]. In addition, as stated in the Tinbergen principle, "if the number of policy targets surpasses the number of instruments, then some targets may not be met" (Tinbergen, 1952).

Ultimately, policymakers should not overburden policy tools with too many targets, as it impedes the proper functioning of that instrument. Following this logic, several developed country central banks have already used balance sheet operations - by altering the size and composition of their balance sheets - to address dysfunctional markets (Bernanke, 2011). The primary contribution of this model is in the addition of a more realistic representation of central bank lending.

2.2.1 Central Bank Lending in DSGE Models

Several papers have been developed with the goal of capturing a more realistic discount window lending function of the central bank in DSGE models. An early contribution is that Gertler and Kiyotaki (2010), who looked to develop a model of discount window lending in a DSGE model. In this setup, as previously mentioned, financial intermediaries are credit constrained and have access to the central bank's lending facilities. However, these commercial banks do not offer collateral in return for central bank liquidity. In a similar fashion, the work of Bocola (2015) builds on the framework of Gertler and Kiyotaki (2010) to determine the effect of LTROs. However, borrowing is still not collateralised in this model.

The model of van der Kwaak (2015) is based on the work of Gertler and Karadi (2011) but restructures the way in which commercial banks are financed. Commercial banks receive

[17] An instrument "is tighter than another if it is more closely linked to the feature it is meant to influence" (Atkeson et al., 2007).

[18] This includes the balance sheet of the central bank, as well as those of financial institutions.

[19] It is generally believed that monetary policy is not effective in 'leaning' against an upswing in the credit cycle and should rather assume an accommodative position to 'clean' after the bubble has burst (White, 2009).

financing through "net worth, deposits and [central bank] liquidity" (van der Kwaak, 2015). His model deviates from Gertler and Karadi (2011) in that obtaining central bank liquidity requires government bonds as collateral, with private sector assets not being eligible. While there are several similarities between the model from this chapter and and that of van der Kwaak (2015), his structure does not include a dimension for the heterogeneous interbank sector and endogenous default.

In order to model central bank lending accurately, I looked to the early work of Reynard and Schabert (2009), and more recently Schabert (2015) and Hörmann and Schabert (2015). Their framework, as was discussed in the section on financial intermediaries, uses a haircut mechanism to facilitate collateralised lending. This approach is also tied to a broader literature that uses haircuts as tools for monetary policy. Some of the relevant readings in this regard are Adrian and Shin (2009), Ashcraft et al. (2011), Cúrdia and Woodford (2011) and Hilberg and Hollmayr (2011). The model presented aims to deliver insight into how agents (primarily financial intermediaries and firms) respond to changes in the size of a central bank's balance sheet. In addition to changes in size, the composition of the balance sheet is also important.

3 Large Scale Asset Purchases

The literature on large-scale asset purchases (LSAPs) identifies several important channels for the transmission of nonstandard policies[20]. One of the primary channels identified in the literature is the portfolio balance channel. As mentioned in the second chapter, asset purchases are often ineffective in New Keynesian DSGE models, as expertly demonstrated in Eggerston and Woodford (2003). Wallace's irrelevance result posits that reserves and government bonds are perfect substitutes, which means that the portfolio balance effects are not observed in the event of strategic asset purchases. Changes to the central bank balance sheet, in terms of size and composition, effectively have no role to play in affecting real variables. As shown by Cúrdia and Woodford (2011), this result holds even in models with demand-side credit frictions where agents believe assets to be perfect substitutes. However, the recent strand of empirical evidence on LSAPs highlights the fact that portfolio balance effects do exist[21] and contribute substantially to changes in long-term rates[22]. Keeping this in mind, it could be fruitful to look at models of asset purchases that have some degree of imperfect asset substitutability.

[20] There is a discussion on this literature in Chapter **??**.

[21] Chapter **??** provides a discussion on the empirical evidence on large-scale asset purchases.

[22] It should be noted that, although there is a fair number of studies that register the portfolio balance effects, there are some who contest its existence - see the discussion in Chapter **??**

3.1 LSAPs in DSGE

3.1.1 'Flattening the Yield Curve'

After the initial shock of the financial crisis, with the policy rate of most advanced economies nearing the ZLB, central banks considered several alternative tools to stimulate economic activity and address dysfunctional financial markets. As a recourse, most central banks initiated asset purchase programs in order to affect longer-term interest rates. Lowering longer-term rates is thought to assist the traditional monetary policy mechanism by reducing private sector borrowing rates (Bernanke and Reinhart, 2004). Broad channels of operation in achieving this goal, as identified in the literature, are the expectations (or signalling) and portfolio balance channels (Woodford, 2012). The model developed in this chapter exploits the transmission mechanism under portfolio rebalancing. In order to generate the impact from LSAPs in a DSGE model, one requires some assumptions on the substitutability of assets. Models that incorporate this channel have to justify why investors value securities used in asset purchases beyond their risk-adjusted payoff (Chodorow-Reich, 2014).

3.1.1.1 Preferred Habitat One of the earliest DSGE models to capture imperfect asset substitution is the model by Andres et al. (2004). In this model the agents have heterogeneous preferences with regard to long-term government bonds, which generate the portfolio balance effects. Models in this tradition are often referred to as 'preferred habitat' models. In this model, the central bank has the power to influence a specific segment of the yield curve through its manipulation of the relative asset supply. Vayanos and Vila (2009) build on the work of Andres et al. (2004) to develop a general equilibrium preferred habitat model with segmented markets. Apart from these two papers, very few dynamic general equilibrium models incorporating central bank asset purchases existed before the crisis. However, the implementation of balance sheet policies during the financial crisis resulted in a surge in the DSGE literature on the impact of large-scale asset purchases.

Chen et al. (2012) estimate the impact of long-term security purchases in a DSGE model with segmented asset markets. Their preferred habitat approach, in the vein of Vayanos and Vila (2009), allows monetary policy to still be effective at the ZLB, with purchases of long-term securities resulting in local supply effects. Transaction costs added to the supply of long-term securities allow LSAPs to operate while the short-term policy rates is fixed at the ZLB, in that it would be able to flatten the yield curve through the reduction of a risk premium (Chen et al., 2012). Asset purchases in a preferred habitat framework are also proposed in the work of Harrison (2012), and Falagiarda and Saia (2013) to study the impact of unconventional policies, with some minor adjustments to each model to answer specific policy questions.

3.1.1.2 Constrained Borrowing Another line of research attempts to look specifically at LSAPs as a form of central bank intermediation, with the work of Gertler and Karadi (2011) among the first to incorporate asset purchases as a monetary policy tool into a DSGE framework. In the process of intermediation, a financial market agent acquires an asset by issuing the relevant counterparty some form of short-term debt (Gertler and Karadi, 2013). In QE1, for example, the central bank acted as an intermediary by providing short-term government debt (borrowed from the Treasury) in return for illiquid assets to failing financial institutions. LSAPs in this type of model matter only when there is a disruption in the process of financial intermediation. Without constraints on borrowing, any premium arising from activity in the asset market will be eliminated through arbitrage. In this setup, LSAPs will be effective only if private intermediaries face borrowing constraints.

Continuing in this vein is the work of Del Negro et al. (2013), who employ the credit market frictions of Kiyotaki and Moore (2012) in a DSGE model. In their model, firms are allowed to invest only a certain proportion of their illiquid assets, known as a resaleability constraint, whereas government bonds are free from any such restrictions. Government bonds are, therefore, more liquid and the constraint generates a liquidity premium on the unaffected asset. Gertler and Karadi (2013) argue that this type of model is not only applicable to credit policy, but also to the purchase of long-term bonds (i.e. quasi-debt management). The argument is that, without any limits to arbitrage, there should be no premium to exploit on either short- or long-term government bonds. With constraints on private intermediaries and financial market frictions that increase the term premium, long-term rates may be reduced by targeted purchases by the central bank (Gertler and Karadi, 2013). Finally, the model of Cahn et al. (2014) builds on the models of Smets and Wouters (2007), Gertler and Kiyotaki (2010), and Gertler and Karadi (2011) to allow for securities of longer maturity in order to assess the impact of the VLTROs as implemented by the ECB.

3.1.2 Targeted Asset Purchases

During times of financial distress, the rationale behind credit easing lies in its ability to affect long-term interest rates. However, another consideration is the scenario where economic activity picks up again after the crisis. One would expect banks to exit the current deleveraging phase and for asset prices to rejoin their upward trajectory. The danger lies in asset prices that grow to exceed their fundamental value, thereby creating an asset price bubble. White (2009) suggests "pre-emptive tightening" of monetary policy to rein in credit cycles and resist credit bubbles. However, it is generally believed that monetary policy is not effective in "leaning" against an upswing in the credit cycle and should rather assume an accommodative position to "clean" up after the bubble has burst. This result is considered part of the Jackson Hole Consensus, whereby it is considered important to focus the attention of monetary policy on low and stable inflation. As argued by Bernanke and Gertler (2000), attempts to influence asset price movements (i.e. 'leaning against the wind') detracts from the pursuit of the inflation objective. In addition, in a recent article Ajello et al. (2015) find that the benefit from increasing the interest rate to counter the build-up of financial imbalances to be negligible.

This type of reaction also distorts the price stability mandate, assigning a dual mandate to interest rate policy. Tinbergen (1952) comments that policymakers should have "one instrument for one goal", while Mundell (1962) states that "policies should be paired with objectives on which they have the most influence". However, mopping up after a bubble has burst is risky in that policymakers cannot be certain that they will be able to clean up in the aftermath, as is evident by the macroeconomic challenges currently faced. In that case it would be sensible to assign balance sheet measures the role of depressing asset values once a bubble is identified[23]. Balance sheet measures have the added advantage that they could target a specific asset of interest, whereas interest rates would affect all asset values[24]. In my model the central bank could plausibly attempt targeted asset purchases of risky securities (longer-term bonds) in an attempt to improve the health of financial institution balance sheets. The next section presents the extension to the model in the previous chapter to include long-term bond purchases.

4 The Model

This section provides an account of the dynamic stochastic general equilibrium (DSGE) model, which draws from the work of Smets and Wouters (2003, 2007), de Walque et al. (2010), Schabert (2015) and Hörmann and Schabert (2015). The model consists of six sectors. Both the household and firm sectors are closely related to the canonical New-Keynesian DSGE formulation of Smets and Wouters (2003, 2007)[25]. One significant difference is that the firm is allowed to default on its loans, as in de Walque et al. (2010). In addition, financial intermediation is included in the banking sector, which consists of two heterogenous banks, both with the option to default on loans (but not on deposits).

The deposit bank receives deposits from households and provides loans to the interbank market, while the merchant bank borrows from the interbank market and issues loans to firms. Merchant banks are also able to hold bonds issued by the government. These bonds serve as collateral in open market operations. The banking sector is similar to that of de Walque et al. (2010) but

[23]Identification of asset bubbles is another complicated matter and will not be addressed in this dissertation

[24]Naturally, purchases of a specific asset class would also impact on other interest rates

[25]In other words, the model combines advancements in real business cycle (RBC) methodology with sticky prices and wages gathered from the New Keynesian framework.

adds a form of collateralised central bank lending in the vein of Schabert (2015) and Hörmann and Schabert (2015). The government can potentially purchase goods, raise lump-sum taxes and issue bonds. I extend this model in the following chapter to include both short- and long-term bonds, with the long-term bonds modelled as perpetuities, as in Chen et al. (2012). The central bank sets the main refinancing rate according to a Taylor-type rule, supplies reserves in exchange for eligible collateral and decides through the haircut mechanism on the size (and potentially composition) of its balance sheet.

4.1 Households

The household sector in this model closely follows that of Smets and Wouters (2003), which consists of a continuum of infinitely-lived households, indexed by $j \in [0,1]$. Households maximise a lifetime utility function given by

$$\max_{\{C_{j,t},N_{j,t},D_{j,t}\}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta^h)^s \left[U^h_{j,t+s}\right] \tag{1}$$

where β^h is a discount factor and the utility function is separable in consumption and labour.

$$U^h_{j,t} = \frac{1}{1-\sigma_c}(C_{j,t} - hC_{j,t-1})^{1-\sigma_c} - \frac{1}{1+\sigma_n}(N_{j,t})^{1+\sigma_n} \tag{2}$$

Utility depends positively on the consumption of j goods, $C_{j,t}$ (relative to an external habit variable $H_t = h \cdot C_{j,t-1}$) and negatively on the labour supply $N_{j,t}$. The coefficient of relative risk aversion is σ_c, which is also known as the inverse of the intertemporal elasticity of substitution. The Frisch elasticity of labour supply is σ_l. Households maximise utility subject to the flow budget constraint,

$$T_t + \frac{D_{j,t}}{R^d_t} + C_{j,t} = w_{j,t}N_{j,t} + A_{j,t} + \frac{D_{j,t-1}}{\pi_t} + T^r_t. \tag{3}$$

The household invests in deposits, $D_{j,t}$ at the risk-free rate of R^d_t and supplies labour at the real wage rate, $w_{j,t}$. The government taxes households in the form of T_t, and the central bank provides seignorage revenue, T^r_t[26]. It is also assumed, as in Smets and Wouters (2003), that state-contingent securities, $A_{j,t}$, insure against idiosyncratic shocks, leaving households "homogenous with respect to consumption and asset holdings" in equilibrium (Christiano et al., 2005).

At this point, it is worth mentioning that several components of the standard DSGE framework

[26] The public sector is not consolidated in this model

are excluded. First, there is no explicit role for money in this model, which is ostensibly the biggest shortcoming in this model. However, the focus of this thesis is not on the cash holdings of households. Extensions of this model will include a cash-in-advance constraint as the preferred method of delivery[27]. Second, investment opportunities are limited to deposits, but could potentially include several other securities and investment vehicles. Third, households are not subject to preference shocks, which are considered part and parcel of the modern modelling approach in the DSGE literature (Smets and Wouters, 2003, 2007). These additions could potentially be added to the framework, but they are not the focus of the analysis and, therefore, are excluded initially.

4.1.1 Consumption and Savings

The objective function (1) is maximised taking into consideration the flow budget constraint (3), which yields the following first-order conditions for consumption and deposit holdings.

$$(\partial C_t) \qquad (C_t - hC_{t-1})^{-\sigma_c} = \lambda_t^h \tag{4}$$

$$(\partial D_t) \qquad \beta^h \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}^h}{\pi_{t+1}} \right) \right] = \frac{\lambda_t^h}{R_t^d} \tag{5}$$

Combining these equations, with $(C_{t+1} - hC_t)^{-\sigma_c} = \lambda_{t+1}^h$, gives the Euler equation as,

$$\beta^h \mathbb{E}_t \left[\frac{(C_{t+1} - hC_t)^{-\sigma_c}}{\pi_{t+1}} \right] = \frac{(C_t - hC_{t-1})^{-\sigma_c}}{R_t^d} \tag{6}$$

The Euler equation encapsulates the intertemporal consumption and saving decisions of the household. In the next two sections I identify the labour supply and wage-setting behaviour of the household, as presented in Fernández-Villaverde and Rubio-Ramírez (2006).

4.1.2 Labour Supply

Households supply homogenous labour to an intermediate labour union. Each household j has monopolistic power over the supply of its labour services (which means it can set its own price in the labour market). The labour union differentiates these labour services (Smets and Wouters, 2003; Brzoza-Brzezina et al., 2011). Aggregate labour demand N_t is given by the

[27] The paper by Schabert (2015) includes a CIA constraint in favour of the popular MIU method. The exclusion of money in this model is briefly discussed in Appendix B.3.

Dixit-Stiglitz aggregator function,

$$N_t = \left(\int_0^1 (N_{j,t})^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta-1}{\eta}} \tag{7}$$

Labour packers buy the differentiated labour from the unions, and package and resell the services to intermediate goods producers. The maximisation problem for the labour packers, who try to maximise the production function, given by (7), is

$$\max_{N_{j,t}} \left(w_t N_t - \int_0^1 w_{j,t} N_{j,t} dj \right) \tag{8}$$

where w_t^h represents the households' differentiated labour wages and w_t, the aggregate wage. The first-order condition for the maximisation problem is

$$w_t \frac{\eta}{\eta - 1} \left(\int_0^1 (N_{j,t})^{\frac{\eta-1}{\eta}} d^j \right)^{\frac{-1}{\eta-1}} \frac{\eta - 1}{\eta} (N_{j,t})^{\frac{-1}{\eta}} - w_{j,t} = 0 \tag{9}$$

and the associated labour demand function is

$$N_{j,t} = \left(\frac{w_{j,t}}{w_t} \right)^{-\eta} N_t \qquad \forall j \tag{10}$$

where the aggregate wage in the economy is represented by

$$w_t = \left(\int_0^1 (w_{j,t})^{1-\eta} dj \right)^{\frac{1}{1-\eta}} \tag{11}$$

4.1.3 Wage Setting

In this economy, the households set their wages according to Calvo's setting. In this scheme, households can optimally adjust their wages after receiving a random signal with probability $(1 - \theta_w)$. A household j that receives this signal will be able to set a new nominal wage to maximise its utility subject to the demand for labour services. Households that do not receive the signal can only partially index their wages to past values of inflation according to the following rule:

$$w_{j,t+1} = (\pi_t)^{\tau_w} w_{j,t} \tag{12}$$

where τ_w is the degree of wage indexation. This implies that if the household cannot change the wage for k periods, with $\tau_w = 0$, then the normalised wage after k periods is $\prod_{s=1}^{k} \frac{(\pi_{t+s-1})^{\tau_w}}{\pi_{t+s}} w_{j,t}$. The maximisation problem relies not only on the optimisation of (1) with respect to the budget constraint in (3), but also on the labour demand function presented in (10) and the wage indexation formula in (12). This relevant part of the maximisation is given by

$$\max_{w_{j,t}} \mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta^h \theta_w\right)^k \left[-\frac{1}{1+\sigma_n} (N_{j,t})^{1+\sigma_n} + \lambda_{j,t+s}^h \prod_{s=1}^{k} \frac{(\pi_{t+s-1})^{\tau_w}}{\pi_{t+s}} w_{j,t} N_{j,t+k} \right] \tag{13}$$

subject to

$$N_{j,t+k} = \left(\prod_{s=1}^{k} \frac{(\pi_{t+s-1})^{\tau_w}}{\pi_{t+s}} \frac{w_{j,t}}{w_{t+k}} \right)^{-\eta} N_{t+k} \quad \forall j \tag{14}$$

All households set the same wage because, as stated in Fernández-Villaverde and Rubio-Ramírez (2006), "complete markets allow them to hedge the risk of the timing of wage change"; this means we drop the j[28]. The first-order condition for this problem is

$$\begin{aligned} &\frac{\eta-1}{\eta} w_t^* \mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta^h \theta_w\right)^k \left[\lambda_{t+s}^h \left(\prod_{s=1}^{k} \frac{(\pi_{t+s-1})^{\tau_w}}{\pi_{t+s}} \right)^{1-\eta} \left(\frac{w_t^*}{w_{t+k}} \right)^{-\eta} N_{t+k} \right] \\ &= \mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta^h \theta_w\right)^k \left[\left(\prod_{s=1}^{k} \frac{(\pi_{t+s-1})^{\tau_w}}{\pi_{t+s}} \frac{w_t^*}{w_{t+k}} \right)^{-\eta(1+\sigma_n)} (N_{t+k})^{1+\sigma_n} \right] \end{aligned} \tag{15}$$

From this we can define

$$f_t^1 = \frac{\eta-1}{\eta} w_t^* \mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta^h \theta_w\right)^k \left[\lambda_{t+s}^h \left(\prod_{s=1}^{k} \frac{(\pi_{t+s-1})^{\tau_w}}{\pi_{t+s}} \right)^{1-\eta} \left(\frac{w_t^*}{w_{t+k}} \right)^{-\eta} N_{t+k} \right] \tag{16}$$

$$f_t^2 = \mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta^h \theta_w\right)^k \left[\left(\prod_{s=1}^{k} \frac{(\pi_{t+s-1})^{\tau_w}}{\pi_{t+s}} \frac{w_t^*}{w_{t+k}} \right)^{-\eta(1+\sigma_n)} (N_{t+k})^{1+\sigma_n} \right] \tag{17}$$

The equality $f_t^1 = f_t^2$ returns the first order condition. It is possible to express f_t^1 and f_t^2 recursively as

$$f_t^1 = \frac{\eta-1}{\eta} (w_t^*)^{1-\eta} \lambda_t^h (w_t)^\eta N_t + \beta^h \theta_w \mathbb{E}_t \left(\frac{(\pi_t)^{\tau_w}}{\pi_{t+1}} \right)^{1-\eta} \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\eta-1} f_{t+1}^1 \tag{18}$$

$$f_t^2 = \left(\frac{w_t}{w_t^*} \right)^{\eta(1+\sigma_n)} (N_t)^{(1+\sigma_n)} + \beta^h \theta_w \mathbb{E}_t \left(\frac{(\pi_t)^{\tau_w}}{\pi_{t+1}} \right)^{-\eta(1+\sigma_n)} \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\eta(1+\sigma_n)} f_{t+1}^2 \tag{19}$$

[28] w_t^* is the common reset price

Since $f_t^1 = f_t^2$, we can set $f_t = f_t^1 = f_t^2$, which gives us

$$f_t = \frac{\eta - 1}{\eta}(w_t^*)^{1-\eta}\lambda_t(w_t)^\eta N_t + \beta^h \theta_w \mathbb{E}_t \left(\frac{(\pi_t)^{\tau_w}}{\pi_{t+1}}\right)^{1-\eta} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta-1} f_{t+1} \tag{20}$$

$$f_t = \left(\frac{w_t}{w_t^*}\right)^{\eta(1+\sigma_n)} (N_t)^{(1+\sigma_n)} + \beta^h \theta_w \mathbb{E}_t \left(\frac{(\pi_t)^{\tau_w}}{\pi_{t+1}}\right)^{-\eta(1+\sigma_n)} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta(1+\sigma_n)} f_{t+1} \tag{21}$$

Finally, given equation (11), the optimal wage-setting problem delivers the following real wage index law of motion

$$w_t^{1-\eta} = \theta_w \left(\frac{(\pi_{t-1})^{\tau w}}{\pi_t}\right)^{1-\eta} (w_{t-1})^{1-\eta} + (1-\theta_w)(w_t{}^*)^{1-\eta} \tag{22}$$

Having defined the first-order conditions that govern the behaviour of the household, I move on to a model of the firm in the next section.

4.2 Firms

Firms in this paper resemble those in the standard New-Keynesian literature, which translates to a single final good and continuum of intermediate goods being produced. In this setting, the final goods sector is perfectly competitive, while one encounters monopolistic competition in the markets for intermediate goods. Intermediate goods are indexed by i, where i is distributed over the unit interval. These firms produce differentiated goods and sell them to aggregators – who combine them into the final good. In other words, final goods producers package the intermediate goods and sell them to households for consumption.

4.2.1 Final-Good Sector

Final goods producers are the aggregators in this economy. They produce a homogenous good Y_t by combining intermediate goods $y_{i,t}$ through a Dixit-Stiglitz technology. The associated production function is,

$$Y_t = \left(\int_0^1 (y_{i,t})^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} \tag{23}$$

where $y_{i,t}$ is the quantity of the intermediate good used in production, ϵ is the elasticity of substitution (time-varying markup in the goods market). Final goods producers maximise their profits subject to the production function in (23), taking as given all intermediate goods prices and the final goods price. The maximisation problem of the final goods producer is

$$\max_{y_{i,t}} \left(p_t Y_t - \int_0^1 p_{i,t} y_{i,t} di \right) \tag{24}$$

The associated input-demand function (same procedure used when calculating the wages) from this problem is

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\epsilon} Y_t \tag{25}$$

and the final goods price is,

$$p_t = \left(\int_0^1 (p_{i,t})^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \tag{26}$$

4.2.2 Intermediate Goods Producers

In this section, there is a continuum of monopolistically competitive intermediate goods producers of unit mass. Firms set their prices, $p_{i,t}$, according to the Rotemberg pricing assumption to maximise profit, π^f[29]. In addition to setting their prices they choose the a level of employment, $N_{i,t}$, and the amount they wish to borrow from merchant banks, $L^b_{i,t}$. These firms also default on their loan repayment, with probability $1 - \psi_t$. In the case of default, firms experience both disutility and pecuniary costs. Combining these elements, one finds that the firm maximises profit in the following manner:

$$\max_{\{p_{i,t}, N_{i,t}, L^b_{i,t}, \psi_t, y_{i,t}, K_{i,t}, \pi^f_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta^f)^s \left[(\pi^f_{t+s}) - d_\psi (1 - \psi_{t+s}) \right] \tag{27}$$

where B^f is the firm's discount factor, and d_ψ is the disutility parameter associated with default. Each good is produced (supplied) using the following Cobb-Douglas production technology:

$$y_{i,t} = K^\alpha_{i,t} N^{1-\alpha}_{i,t} \tag{28}$$

[29]The Rotemberg pricing mechanism is used because interest rates are predetermined, which means that marginal cost is unique to the firm. I did not want price setting to interfere with the default decision which meant that the symmetry of Rotemberg wourld be most appropriate, see de Walque and Pierrard (2010) for a discussion. Rotemberg and Calvo present with the same reduced-form New Keynesian Phillips Curve, representing similar dynamics for inflation and output. In other words, up to the first order, the dynamics of these two mechanisms are the same (Blanchard and Galí, 2007).

where $K_{i,t}$ is the capital rented by the firm and $N_{i,t}$ the number of workers employed (i.e. labour input rented). Capital accumulation for this firm is characterised by

$$K_{i,t} = (1-\varphi)K_{i,t-1} + \frac{L^b_{i,t}}{R^c_t}\left[1 - \Gamma\left(\frac{L^b_{i,t}}{L^b_{i,t-1}}\right)\right] \tag{29}$$

where φ is the rate at which capital depreciates and Γ is a convex investment adjustment cost[30]. In this equation firms replenish their capital stock by borrowing $L^b_{i,t}$ at a price of $\frac{1}{R^c_t}$. Finally the profit function is given by,

$$\begin{aligned} \pi^f_t = \left(\frac{p_{i,t}}{p_t}\right) y_{i,t} - w_t N_{i,t} - \psi_t \frac{L^b_{i,t-1}}{\pi_t} - \frac{\omega_\psi}{2}\left((1-\psi_{t-1})L^b_{i,t-2}\right)^2 \\ -\frac{\varrho}{2}\left(\frac{p_{i,t}}{(\bar{\pi})^{1-\gamma_p}(\pi_{t-1})^{\gamma_p} p_{i,t-1}} - 1\right)^2 Y_t \end{aligned} \tag{30}$$

where ω_ψ is the pecuniary cost of default parameter, ϱ is a quadratic price adjustment cost and $\bar{\pi}$ is the steady-state value of inflation. The FOCs with respect to capital and labour for this problem are

$$(\partial N_{i,t}) \qquad w_t = (1-\alpha)K^\alpha_{i,t} {L_{i,t}}^{-\alpha} \tag{31}$$

$$(\partial K_{i,t}) \qquad \lambda^f_{i,t} - \beta^f \mathbb{E}_t[(1-\varphi)\lambda^f_{i,t+1}] = \alpha K^{\alpha-1}_{i,t} L^{1-\alpha}_{i,t} \tag{32}$$

which means that the constant returns-to-scale Cobb-Douglas production function delivers the following marginal cost function:

$$\begin{aligned} mc_t &= \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t}{\alpha}\right)^\alpha \\ &= \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{\lambda^f_t - \beta^f \mathbb{E}_t[(1-\varphi)\lambda^f_{t+1}]}{\alpha}\right)^\alpha \end{aligned} \tag{33}$$

[30] This convex investment adjustment cost function $\Gamma(\cdot)$ is equal to zero at steady state. In addition, the first derivative $\Gamma'(\cdot)$ also equals zero at steady state. The explicit functional form is $\Gamma\left(\frac{L^b_{i,t}}{L^b_{i,t-1}}\right) = \frac{\theta}{2}\left(\frac{L^b_{i,t}}{L^b_{i,t-1}} - 1\right)^2$.

The investment equation derived from the first-order conditions for this maximisation problem is

$$
\begin{aligned}
(\partial L^b_{i,t}) \qquad & \frac{\lambda^f_{i,t}}{R^c_t}\left(1-\Gamma\left(\frac{L^b_{i,t}}{L^b_{i,t-1}}\right)-\Gamma'\left(\frac{L^b_{i,t}}{L^b_{i,t-1}}\right)\frac{L^b_{i,t}}{L^b_{i,t-1}}\right)\\
&=\beta^f\mathbb{E}_t\left[\frac{\psi_{t+1}}{\pi_{t+1}}-\frac{\lambda^f_{i,t+1}}{R^c_{t+1}}\left(\Gamma'\left[\frac{L^b_{i,t+1}}{L^b_{i,t}}\right]\left(\frac{L^b_{i,t+1}}{L^b_{i,t}}\right)^2\right)\right]+(\beta^f)^2\mathbb{E}_t\left[\omega_\psi(1-\psi_{t+1})^2L^b_{i,t}\right]
\end{aligned}
\tag{34}
$$

The default decision is reflected by

$$
(\partial\psi_t) \qquad \frac{L^b_{i,t-1}}{\pi_t}=d_\psi+\beta^f\omega_\psi\left[(1-\psi_t)(L^b_{i,t-1})^2\right] \tag{35}
$$

4.2.2.1 Price setting I used the Rotemberg pricing assumption, so that all intermediate firms set the same prices and produce the same quantities (de Walque et al., 2010). The price was set by taking into account the marginal cost, price adjustment cost and the market demand function. The relevant part of the maximisation problem is as follows:

$$
\max_{\{p_{i,t}\}}=\mathbb{E}_t\sum_{s=0}^{\infty}(\beta^f)^s\left[\left(\frac{p_{i,t}}{p_t}\right)y_{i,t}-(mc_t)y_{i,t}-\frac{\varrho}{2}\left(\frac{p_{i,t}}{(\bar\pi)^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}p_{i,t-1}}-1\right)^2Y_t\right] \tag{36}
$$

subject to the demand for intermediate goods,

$$
y_{i,t}=\left(\frac{p_{i,t}}{p_t}\right)^{-\epsilon}Y_t
$$

Inserting the value for $y_{i,t}$ gives the following problem:

$$
\max_{\{p_{i,t}\}}\mathbb{E}_t\sum_{s=0}^{\infty}(\beta^f)^s\left[\left(\frac{p_{i,t}}{p_t}\right)\left(\frac{p_{i,t}}{p_t}\right)^{-\epsilon}Y_t-(mc_t)\left(\frac{p_{i,t}}{p_t}\right)^{-\epsilon}Y_t-\frac{\varrho}{2}\left(\frac{p_{i,t}}{(\bar\pi)^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}p_{i,t-1}}-1\right)^2Y_t\right]
$$

The first-order condition with respect to $p_{i,t}$ is,

$$
\begin{aligned}
(\partial p_{i,t}) \qquad & (1-\epsilon)(p_{i,t})^{-\epsilon}(p_t)^{\epsilon-1}Y_t+(\epsilon)mc_t(p_{i,t})^{-\epsilon-1}(p_t)^{\epsilon}Y_t\\
&-\varrho Y_t\left(\frac{p_{i,t}}{(\bar\pi)^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}p_{i,t-1}}-1\right)\left(\frac{1}{(\bar\pi)^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}p_{i,t-1}}\right)\\
&+\varrho\beta^f\mathbb{E}_tY_{t+1}\left(\frac{p_{i,t+1}}{(\bar\pi)^{1-\gamma_p}(\pi_t)^{\gamma_p}p_{i,t}}-1\right)\left(\frac{p_{i,t+1}}{(\bar\pi)^{1-\gamma_p}(\pi_t)^{\gamma_p}(p_{i,t})^2}\right)
\end{aligned}
$$

Aggregate over all retailer prices, i.e. $p_t = \int_0^1 (p_{i,t})di$, gives us the following price Phillips curve:

$$\left(\frac{\pi_t}{(\bar{\pi})^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}} - 1\right)\frac{\pi_t}{(\bar{\pi})^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}}$$
$$= \beta^f \mathbb{E}_t \left[\left(\frac{\pi_{t+1}}{(\bar{\pi})^{1-\gamma_p}(\pi_t)^{\gamma_p}} - 1\right)\frac{\pi_{t+1}}{(\bar{\pi})^{1-\gamma_p}(\pi_t)^{\gamma_p}}\frac{y_{t+1}}{y_t}\right] + \left[\frac{1-\epsilon(1+mc_t)}{\varrho}\right] \tag{37}$$

4.3 Banking Sector

The banking sector consists of two specialised banks. Deposit banks receive deposits from households at the deposit rate and lend money to the interbank market at the interbank rate. The merchant bank is the link to the firm; it borrows from the interbank market and supplies loans to the firms. Both of these banks may face defaults on their loans.

4.3.1 Deposit Banks

Deposit banks lend L_t^l to the interbank market at the interbank rate R_t^l. It is possible, with probability $(1-\delta_t)$, that the bank is not reimbursed for its loan. These banks also receive deposits $D_t^l = \int D_{j,t}dj$ from households, which they must pay at a deposit rate of R_t^d. There is no possibility for deposit banks to default on the loans of households. The maximisation programme for the bank is

$$\max_{\{D_t^l, L_t^l\}} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta^l)^s \left[\frac{(\pi_{t+s}^l + 1)^{1-\sigma_l}}{1-\sigma_l}\right] \tag{38}$$

subject to the constraint

$$\pi_t^l = \frac{D_t^l}{R_t^d} - \frac{D_{t-1}^l}{\pi_t} + \delta_t \frac{L_{t-1}^l}{\pi_t} - \frac{L_t^l}{R_t^l} \tag{39}$$

The formulation of the bank in this model abstracts from real security holdings[31], a supervisory authority and own funds, when compared with the setup in de Walque et al. (2010). The balanced budget condition here is $D_t^l = L_t^l$. The most important contribution to the paper lies

[31]Future versions of this model will include an array of security options

with the merchant banks. The first-order conditions for this maximisation problem are,

$$(\partial D_t^l) \qquad \frac{1}{R_t^d}\dot{U}_t^l = \beta^l \mathbb{E}_t\left[\frac{1}{\pi_{t+1}}\dot{U}_{t+1}^l\right] - \Xi_t \tag{40}$$

$$(\partial L_t^l) \qquad \frac{1}{R_t^l}\dot{U}_t^l = \beta^l \mathbb{E}_t\left[\frac{\delta_{t+1}}{\pi_{t+1}}\dot{U}_{t+1}^l\right] + \Xi_t \tag{41}$$

where $\dot{U}_t^l = (\pi_t^l + 1)^{-\sigma_l}$ and Ξ_t is the multiplier on the balanced budget condition. These first-order conditions are the Euler equations for deposits from households and loans to the interbank market, respectively (de Walque et al., 2010). Combining them gives the following equation:

$$\frac{1}{R_t^d}\dot{U}_t^l = \beta^l \mathbb{E}_t\left[\frac{1}{\pi_{t+1}}\dot{U}_{t+1}^l\right] + \beta^l \mathbb{E}_t\left[\frac{\delta_{t+1}}{\pi_{t+1}}\dot{U}_{t+1}^l\right] - \frac{1}{R_t^l}\dot{U}_t^l$$

$$\dot{U}_t^l\left(\frac{1}{R_t^d} + \frac{1}{R_t^l}\right) = \beta^l \mathbb{E}_t\left[\left(\frac{1+\delta_{t+1}}{\pi_{t+1}}\right)\dot{U}_{t+1}^l\right] \tag{42}$$

4.3.2 Merchant Banks

Merchant banks complete the financial intermediation narrative for the banking sector. These banks, referred to by Goodhart et al. (2006) as retail banks, borrow money from the interbank market L_t^l at the interbank rate R_t^l, receive government debt B_t (which is issued at the price $1/R_t^b$ and delivers a payoff of one in $t+1$) and receive reserves from the central bank M_t at the nominal policy rate R_t^m. Merchant banks may choose not to repay their debt, with a probability of $(1-\delta_t)$. Default takes on a very specific character in this model. Banks that default are not excluded from the interbank market. They experience disutility (d_δ) and non-pecuniary costs as a result of this decision. These banks are also the originators of loans for the firms. They provide loans L_t^b at the credit rate of R_t^c. Firms may also choose not to repay their debt, with probability $(1-\alpha_t)$.

The reserves received, M_t, are from outright purchases of securities by the central bank (i.e. permanent open market operations). The monetary authority can perform this operation to change the size of its balance sheet (Akhtar, 1997). It is also possible to include temporary repurchase agreements, as presented in Schabert (2015). These repurchase agreements entail overnight transactions in the reserve market so that the overnight rate is kept in line with the policy rate. This type of fine-tuning is not included in this model.

4.3.2.1 Collateralised Lending It is worthwhile taking some time to explain the mechanism underlying outright purchases. The collateralised lending in this model of Schabert (2015) uses

"Treasury bills as collateral for central bank money", with the price of these liabilities being the policy rate R_t^m. Reserves can be gained, at this policy rate (or repo rate) by exchanging treasuries, ΔB_t^c, in the following way[32]:

$$M_t = \kappa \cdot \Delta \frac{B_t^c}{R_t^m} \quad \text{where} \quad \Delta B_t^c \leq B_{t-1} \tag{43}$$

The second part of this equation relays the fact that the merchant bank is limited to the amount of treasuries exchanged for reserves by the stock of treasuries carried over from the previous period. Following Schabert (2015), I then combined these equations and introduced the collateralised lending constraint on the merchant bank, in the form of

$$M_t \leq \kappa \cdot \frac{B_{t-1}}{R_t^m \pi_t} \tag{44}$$

where $M_t = M_t^p - \frac{M_{t-1}^p}{\pi_t}$. The role of this constraint is embodied in the ability of the merchant bank to "acquire money, M_t, in exchange for the discounted value of treasury bills carried over from the previous period" as elucidated by Bredemeir et al. (2015). Alternatively, this equation could be interpreted as a central bank money supply constraint. In this equation, κ_t represents a monetary policy instrument in addition to the policy rate. This instrument allows the central bank to control the supply of reserves as a fraction of the discounted market value of government bonds held by the private sector

4.3.2.2 Defining κ Before continuing, it is important to conceptualise the meaning of κ, as it is central to the dynamics of this model. This parameter[33] represents different values for the loan-to-value (LTV) ratio, ranging from $0 < \kappa \leq 1$. It is often referred to as the haircut parameter, where a haircut is represented by $(1 - \kappa)$. Finally, it could also be called the collateral requirement. In this thesis it will be referred to either as the haircut parameter/variable or as the collateral requirement. This is done in order to avoid confusion with the LTV ratios of macroprudential policy from the Basel Accords.

The lower values of κ represent a higher required amount of bonds to acquire reserves. As an example, if the value of the parameter is 0.9, then it represents a 10% haircut[34]. To further illustrate this point, consider a treasury security with a value of $100 where the haircut on this

[32] I followed the nomenclature of Schabert (2015) and Bredemeir et al. (2015) in this regard.

[33] Important is important to note that I initially defined κ as a parameter, where the model of Schabert (2015) considers it only as a variable. Defining it as a parameter allows the setting of specific scenarios relevant to the central question of the chapter. I did utilise it in its capacity as a variable, following an AR(1) process, in some scenarios. This is discussed in more detail in the results section.

[34] The naming convention can be somewhat counter-intuitive. I ideally would have wanted to refer to κ as the LTV instead of the haircut (the two are complements).

relatively safe security is 10%. This security would then be accepted as collateral for a loan up to the value of $90, while a riskier security with a higher haircut is only eligible for a relatively smaller loan (Hilberg and Hollmayr, 2011). Increases in the parameter represent an increase in the size of the central bank balance sheet, as the collateral can be used to obtain greater amounts of liquidity.

4.3.2.3 Budget Constraint Finally, merchant banks are subject to the constraint that liabilities equal assets, i.e. liquidity borrowed from the interbank market should equal the expected payoffs from assets. This means that in this model there is a balanced budget constraint, as in Cúrdia and Woodford (2011) and Christoffel and Schabert (2014), which is the following:

$$L_t^l = M_t^p + B_t + L_t^b \tag{45}$$

The balance sheet constraint means that merchant banks need to pay for the interbank liquidity that they obtain through money holdings, bonds or the extension of loans. Another potentially useful constraint to generate demand for money is a cash-in-advance style minimum reserve requirement, such as $\Theta L_{t-1}^l \leq M_t^p$. This was originally included in the model, but its inclusion did not alter the results significantly. A further discussion on this reserve requirement is included in Appendix B.3.

The banks aim to maximise the present value of profits subject to (44), (45) and the profit equation (47). The maximisation is also subject to a disutility cost in d_δ,

$$\max_{\{M_t, M_t^P, B_t, L_t^l, L_t^b, \delta t\}} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta^b)^s \left[\left(\frac{(\pi_{t+s}^b + 1)^{1-\sigma_b}}{1-\sigma_b} \right) - d_\delta (1 - \delta_{t+s}) \right] \tag{46}$$

The real profits of the merchant bank π_t^b is then given by

$$\begin{aligned} \pi_t^b = & \frac{L_t^l}{R_t^l} - \delta_t \frac{L_{t-1}^l}{\pi_t} + \psi_t \frac{L_{t-1}^b}{\pi_t} - \frac{L_t^b}{R_t^c} + \frac{B_{t-1}}{\pi_t} - \frac{B_t}{R_t^b} \\ & - M_t^p + \frac{M_{t-1}^p}{\pi_t} - \frac{\omega_\delta}{2} \left[(1 - \delta_{t-1}) L_{t-2}^l \right]^2 - M_t (R_t^m - 1) \end{aligned} \tag{47}$$

where ω_b represents a pecuniary cost from defaulting. The first-order conditions for this maximisation problem are,

$$
\begin{aligned}
(\partial M_t^P) \qquad & \dot{U}_t^b = \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b}{\pi_{t+1}} \right) - \Upsilon_t \\
(\partial M_t) \qquad & \dot{U}_t^b = R_t^m (\dot{U}_t^b + \eta_t) \\
(\partial B_t) \qquad & \dot{U}_t^b \frac{1}{R_t^b} = \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b + \kappa \cdot \eta_{t+1}}{\pi_{t+1}} \right) - \Upsilon_t \\
(\partial L_t^l) \qquad & \dot{U}_t^b \frac{1}{R_t^l} = \beta^b \mathbb{E}_t \left[\left(\frac{\delta_{t+1}}{\pi_{t+1}} \right) \dot{U}_{t+1}^b \right] + (\beta^b)^2 \mathbb{E}_{t+1} \left[\left(\omega_\delta (1 - \delta_{t+1})^2 L_t^l \right) \dot{U}_{t+2}^b \right] + \Upsilon_t \\
(\partial L_t^b) \qquad & \dot{U}_t^b \frac{1}{R_t^c} = \beta^b \mathbb{E}_t \left[\frac{\psi_{t+1}}{\pi_{t+1}} \dot{U}_{t+1}^b \right] - \Upsilon_t \\
(\partial \delta_t) \qquad & \dot{U}_t^b \frac{L_{t-1}^l}{\pi_t} = d_\delta + \omega_\delta \beta^b \mathbb{E}_t \left[\left((1 - \delta_t)(L_{t-1}^l)^2 \right) \dot{U}_{t+1}^b \right]
\end{aligned}
$$

where $\dot{U}_t^b = (\pi_t^b + 1)^{-\sigma_b}$, η_t is the multiplier associated with the collateralised lending constraint and Υ_t is the multiplier on the balanced budget constraint. For the latter, it can be defined as $\Upsilon_t = \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b}{\pi_{t+1}} \right) - \dot{U}_t^b$. Substituting this into the above equations to eliminate it gives the following set of equations,

$$\dot{U}_t^b = R_t^m (\dot{U}_t^b + \eta_t) \tag{48}$$

$$\frac{1}{R_t^b} = 1 + (\dot{U}_t^b)^{-1} \beta^b \mathbb{E}_t \left(\frac{\kappa \cdot \eta_{t+1}}{\pi_{t+1}} \right) \tag{49}$$

$$\frac{1}{R_t^l} = 1 - (\dot{U}_t^b)^{-1} \beta^b \mathbb{E}_t \left[\left(\frac{\delta_{t+1} + 1}{\pi_{t+1}} \right) \dot{U}_{t+1}^b \right] + (\dot{U}_t^b)^{-1} (\beta^b)^2 \mathbb{E}_{t+1} \left[\left(\omega_\delta (1 - \delta_{t+1})^2 L_t^l \right) \dot{U}_{t+2}^b \right] \tag{50}$$

$$\frac{1}{R_t^c} = 1 + (\dot{U}_t^b)^{-1} \beta^b \mathbb{E}_t \left[\left(\frac{\psi_{t+1} - 1}{\pi_{t+1}} \right) \dot{U}_{t+1}^b \right] \tag{51}$$

$$\dot{U}_t^b \frac{L_{t-1}^l}{\pi_t} = d_\delta + \omega_\delta \beta^b \mathbb{E}_t \left[\left((1 - \delta_t)(L_{t-1}^l)^2 \right) \dot{U}_{t+1}^b \right] \tag{52}$$

The following complimentary slackness condition holds:

$$\eta_t \left[\kappa \frac{B_{t-1}}{\pi_t} - R_t^m M_t \right] = 0, \quad \eta_t \geq 0, \quad \left(\kappa \frac{B_{t-1}}{\pi_t} - R_t^m M_t \right) \geq 0$$

4.4 Public Sector

4.4.1 Government

The government in this economy buys goods, has access to lump-sum transfers T_t and issues debt (in the form of bonds). Bonds are held by either the merchant banks B_t or the central bank B_t^c. The total stock of newly issued bonds by the government is $B_t^g = B_t + B_t^c$. Following Reynard and Schabert (2009) and Schabert (2015) the growth of the bond supply is constant, equal to Ω and exogenously determined. It is given by

$$B_t^g = \Omega B_{t-1}^g \tag{53}$$

where $\Omega > 1$. As shown with the calibration, the growth of these short-term securities is close to that of inflation. Only short-term risk-free bonds, similar to treasury bills, are considered in this chapter, with bonds of longer maturity introduced in the next chapter. The budget constraint for the treasury is

$$G_t + \frac{B_t^g}{R_t^b} = \frac{B_{t-1}^g}{\pi_t} + T_t \tag{54}$$

4.4.2 Central Bank

The monetary authority is able to supply reserves outright through open market purchases, where newly issued reserves is reflected by, $M_t = M_t^p - \frac{M_{t-1}^p}{\pi_t}$. The central bank collects government bonds in return for newly issued reserves. In addition, interest accrues at the main refinancing rate R_t^m, which translates into a return of $M_t{\cdot}R_t^m$ at period t.

The budget constraint for the central bank, following Schabert (2015) is,

$$T_t^r - M_t R_t^m = \frac{B_{t-1}^c}{\pi_t} - \frac{B_t^c}{R_t^b} \tag{55}$$

Following Reynard and Schabert (2009) and Bredemeir et al. (2015), seignorage revenues are presented as

$$T_t^r = B_t^c \left(1 - \frac{1}{R_t^b}\right) + (R_t^m - 1) M_t \tag{56}$$

The public sector is not consolidated, and the central bank transfers go directly to households. When I substituted the central bank transfers, as in the central bank budget constraint, bond holdings evolved according to

$$B_t^c - \frac{B_{t-1}^c}{\pi_t} = M_t^p - \frac{M_{t-1}^p}{\pi_t} \tag{57}$$

Following Schabert (2015), I restricted the initial values, $B_{-1}^c = M_{-1}^p$, which leads to a central bank balance sheet condition, with $B_t^c = M_t^p$. In addition, the central bank sets the policy rate according to a feedback rule, which takes into account how the central bank adjusts the p olicy rate response to changes in its own lags, inflation, a measure for the real output-gap, and contemporary output growth:

$$R_t^m = (R_{t-1}^m)^{\rho_r}(R^m)^{1-\rho_r}\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi(1-\rho_r)}\left(\frac{y_t}{y}\right)^{\rho_y(1-\rho_r)}\left(\frac{y_t}{y_{t-1}}\right)^{\rho_d y(1-\rho_r)} \tag{58}$$

To summarise, in this model the central bank has two instruments. As delineated by Hörmann and Schabert (2013), these instruments are presented as the following:

1. *Conventional instrument*: The policy rate, R_t^m.
2. *Quantitative easing (size of balance sheet)*: Increase reserves available against eligible assets (short-term bonds) in open market operations by increasing κ.

Quantitative easing in this setting consists only of changes in the size of the balance sheet, although the next chapter includes a dimension for the composition. Selecting a value for κ determines the size, with larger values resulting in a greater selection of bonds available as collateral.

4.5 Market Clearing/Aggregation

The final goods market is in equilibrium. Firms and banks directly consume their profits (not owned by households). The aggregate resource constraint is

$$\begin{aligned} Y_t = {} & C_t + G_t + \pi_t^f + \pi_t^b + \pi_t^l + K_t - (1-\varphi)K_{t-1} + \frac{L_t^b}{R_t^c}\left[\Gamma\left(\frac{L_t^b}{L_{t-1}^b}\right)\right] \\ & + \frac{\omega_\delta}{2}\left[(1-\delta_{t-1})L_{t-2}^l\right]^2 + \frac{\omega_\psi}{2}\left[(1-\psi_{t-1})L_{t-2}^b\right]^2 \\ & + \frac{\varrho}{2}\left(\frac{p_t}{(\bar{\pi})^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}p_{t-1}} - 1\right)^2 Y_t \end{aligned} \tag{59}$$

The aggregate production function is

$$Y_t = K_t^{\alpha} N_{i,t}^{1-\alpha} \tag{60}$$

The aggregated law of motion for capital is

$$K_t = (1-\varphi)K_{t-1} + \frac{L_t^b}{R_t^c}\left[1 - \Gamma\left(\frac{L_t^b}{L_{t-1}^b}\right)\right] \tag{61}$$

5 The Model

The model presented here gives the central bank the ability to influence the supply of long-term government bonds. In particular, it combines elements of the papers by Chen et al. (2012), Niestroj et al. (2013), Hörmann and Schabert (2015), and Schabert (2015). In this section the amendments to the model in Chapter **??** are presented, with only the affected sectors shown. Of particular importance are the changes to the merchant bank sector, which gains a new investment vehicle (namely long-term securities). Imperfect asset substitutability is enforced by the collateralised borrowing mechanism. The irrelevance result is overcome through the introduction of multiple assets that are viable as collateral for reserves (Niestroj et al., 2013). A liquidity premium is generated through the interest rate spread on eligible versus non-eligible assets. This embedded premium can be manipulated by the central bank through its setting of the relevant haircut parameter (Schabert, 2015). Government and central bank sectors are also implicated by the introduction of these bonds.

5.1 Merchant Bank

The following section describes the behaviour of merchant banks, with the important introduction of long-term securities in both the budget and collateral constraints. I have chosen to adopt the approach of Woodford (1998, 2001) in modelling the stock of long-term bonds. This method is widely accepted and is also used in the work of Andres et al. (2004), Chen et al. (2012), Harrison (2012), van der Kwaak (2015), and Chin et al. (2015).

5.1.1 Long-term Bonds

In this setting government bonds are modelled as perpetuities[35] that cost p_t^L at time t and pay an exponentially decaying coupon Φ^s at time $t+s+1$, where $0 < \Phi \leq 1$. Introducing long-term bonds into the profit function (47) from Chapter **??** results in the following addition:

$$\pi_t^b = \ldots + \sum_{s=1}^{\infty} \Phi^{s-1} B_{t-s}^L - p_t^L B_t^L \ \ldots \tag{62}$$

In order to simplify this equation, one can take advantage of the fact that the period t price of a long-term bond issued s periods ago, p_{t-s}^L, in period $t-s$, is a function of the coupon and current price of the bond, as pointed out by Woodford (2001) and Chen et al. (2012). In other words

$$p_{t-s}^L = \Phi^s p_t^L \tag{63}$$

Using this equation, the profit equation (62) can be rewritten in a "recursive formulation", as in Chen et al. (2012). In other words, the equation is used to transform $\sum_{s=1}^{\infty} \Phi^{s-1} B_{t-s}^L$ into a function of B_t^L. This simplification allows long-term bonds to be expressed as one-period bonds that pay their nominal return after one period, similar to the way in which short-term bonds are represented.

A long-term bond issued $s-1$ periods ago is equal to Φ^{s-1} new bonds. At time $t-1$, with no arbitrage, using (63) delivers the following:

$$\begin{aligned}
p_{t-1}^L B_{t-1}^L &= \sum_{s=1}^{\infty} p_{t-s}^L B_{t-s}^L \qquad \text{substitute } p_{t-s}^L \text{ with } p_{t-1}^L \Phi^{s-1} \\
p_{t-1}^L B_{t-1}^L &= \sum_{s=1}^{\infty} p_{t-1}^L \Phi^{s-1} B_{t-s}^L \qquad \text{divide by } p_{t-1}^L \\
B_{t-1}^L &= \sum_{s=1}^{\infty} \Phi^{s-1} B_{t-s}^L
\end{aligned}$$

In addition, as given in Chen et al. (2012), the gross yield to maturity (long-term interest rate) on the long-term bond is,

$$R_t^L = \frac{1}{p_t^L} + \Phi^s \tag{64}$$

[35]As explained in Woodford (2001), this perpetuity generates a constant, infinite, stream of interest payments. A practical example of this is the British consol. An interesting paper by Cochrane (2015) suggests using perpetuities in dealing with US Federal debt.

Perpetuity redeems all old bonds in each period, where the bond pays the following:

$$1 + \Phi p_{t+1}^L = p_{t+1}^L \left(\frac{1}{p_{t+1}^L} + \Phi^s \right) = p_{t+1}^L R_{t+1}^L \tag{65}$$

At time t we have that B_{t-1}^L is worth $B_{t-1}^L(1 + \Phi p_t^L)$. Substituting it with p_t^L we get, at time t, that B_{t-1}^L is worth $B_{t-1}^L(1 + [\Phi/(R_t^L - \Phi)]) = p_t^L R_t^L B_{t-1}^L$.

This alters the real profits of the merchant bank in the following way, giving the final form

$$\begin{aligned} \pi_t^b = & \frac{L_t^l}{R_t^l} - \delta_t \frac{L_{t-1}^l}{\pi_t} + \psi_t \frac{L_{t-1}^b}{\pi_t} - \frac{L_t^b}{R_t^c} + \frac{B_{t-1}^S}{\pi_t} - \frac{B_t^S}{R_t^b} + \frac{p_t^L R_t^L B_{t-1}^L}{\pi_t} - p_t^L B_t^L \\ & - M_t^p + \frac{M_{t-1}^p}{\pi_t} - \frac{\omega_\delta}{2} \left[(1 - \delta_{t-1}) L_{t-2}^l \right]^2 - M_t (R_t^m - 1) \end{aligned} \tag{66}$$

In addition to the change on the budget constraint, the introduction of long-term bonds impacts the open market operation mechanism, adding another dimension to policy action. Now we are not concerned only with changing the size of the balance sheet; we wish to change the composition as well. This means that our collateral constraint changes to become

$$M_t \leq \kappa^s \cdot \frac{B_{t-1}^S}{R_t^m \pi_t} + \kappa^l \cdot \frac{p_t^L R_t^L B_{t-1}^L}{R_t^m \pi_t} \tag{67}$$

An increase in both κ^s and κ^l will result in a change in the size and composition of the balance sheet of the bank. However, if there is a change in κ^l, with a corresponding sterilising change in κ^s, then the size will remain the same but the composition will change (Hörmann and Schabert, 2015). The central bank, in this regard, has full control over its balance sheet, providing intermediation services to merchant banks (Niestroj et al., 2013). Effective demand for securities of differing maturities is determined by the exogenous combination of κ values.

The balanced budget condition also changes to incorporate the newly introduced security. It now looks similar to that of Niestroj et al. (2013). This equation is given by

$$L_t^l = M_t^p + L_t^b + B_t^S + \mathbb{E}_t \left(p_{t+1}^L R_{t+1}^L B_t^L \right) \tag{68}$$

The first-order conditions for the merchant bank remain the same, with the exception of an additional derivative with respect to long-term bonds. The new FOC is

$$(\partial B_t^L) \qquad \dot{U}_t^b p_t^L = \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b + \kappa^l \cdot \eta_{t+1} p_{t+1}^L R_{t+1}^L}{\pi_{t+1}} \right) - \Upsilon_t \mathbb{E}_t \left(p_{t+1}^L R_{t+1}^L \right) \tag{69}$$

The complimentary slackness condition for the merchant bank now becomes

$$\eta_t \left[\kappa^s \cdot \frac{B_{t-1}^S}{\pi_t} + \kappa^l \cdot \frac{p_t^L R_t^L B_{t-1}^L}{\pi_t} - R_t^m M_t \right] = 0$$
$$\eta_t \geq 0, \quad \left(\kappa^s \cdot \frac{B_{t-1}^S}{\pi_t} + \kappa^l \cdot \frac{p_t^L R_t^L B_{t-1}^L}{\pi_t} - R_t^m M_t \right) \geq 0$$

5.2 Public Sector

5.2.1 Government

In general, the public sector consists of a treasury (government) and a central bank. The government performs the same functions discussed in the previous chapter. The exception introduced in this chapter is that of its issuance of long-term bonds. Short-term bonds still grow at a constant rate, where $\Gamma \geq 1$. Long-term bonds, however, are modelled as perpetuities (as alluded to in the section on the merchant bank). The growth of long-term bonds follows an autoregressive process, as in Chen et al. (2012),

$$p_t^L B_t^L = \left(\frac{p_{t-1}^L B_{t-1}^L}{\pi_t} \right)^{\rho_b} e^{\xi_t^L} \tag{70}$$

where $\rho_b \in (0, 1)$ and $\xi_{L,t}$ is an i.i.d. exogenous shock. Similar to Chapter 4.4.1, B_t^{TS} is the total stock of short-term bonds, which is held either in the banking sector B_t^S or by the central bank, B_t^{CS}. The equation that represents this relation is $B_t^{TS} = B_t^S + B_t^{CS}$. In a similar vein, we will have that the total supply of long-term bonds is $B_t^{TL} = B_t^L + B_t^{CL}$. Introducing long-term bonds changes the budget constraint to read

$$G_t + \frac{B_{t-1}^{TS}}{\pi_t} + \frac{p_t^L R_t^L B_{t-1}^{TL}}{\pi_t} = \frac{B_t^{TS}}{R_t^b} + p_t^L B_t^{TL} + T_t \tag{71}$$

The left-hand side of the equation represents spending in real terms, while the right represents treasury income. The first term on the left-hand side represents real government spending, while the other components are the real cost of servicing bonds that are maturing in the current period. On the right-hand side, we have the market value of the total amount of short- and long-term bonds issued in the current period, in addition to tax collected (Chen et al., 2012).

5.2.2 Central Bank

The monetary authority is able to supply money outright through open market purchases (M_t^p). The central bank collects government bonds in return for newly issued money. In addition, interest accrues at the main refinancing rate, R_t^m, which translates into a return of $M_t \cdot R_t^m$ at period t. Newly issued money is reflected by, $M_t = M_t^p - \frac{M_{t-1}^p}{\pi_t}$, for which the central bank receives government bonds.

The interest earnings of the central bank, as in Schabert (2015), is

$$T_t^r = B_t^{CS}\left(1 - \frac{1}{R_t^b}\right) + \frac{p_t^L R_t^L B_{t-1}^{CL}}{\pi_t} - p_{t-1}^L B_{t-1}^{CL} + M_t(R_t^m - 1) \tag{72}$$

The central bank reinvests the wealth exclusively in government bonds, which means the budget constraint adjusted for long-term bonds now becomes

$$T_t^r - \frac{B_{t-1}^{CS}}{\pi_t} + \frac{B_t^{CS}}{R_t^b} = \frac{p_t^L R_t^L B_{t-1}^{CL}}{\pi_t} - p_t^L B_t^{CL} + \left(M_t^p - \frac{M_{t-1}^p}{\pi_t}\right) R_t^m \tag{73}$$

Bond holdings evolve according to

$$B_t^{CS} - \frac{B_{t-1}^{CS}}{\pi_t} + p_t^L B_t^{CL} - \frac{p_{t-1}^L B_{t-1}^{CL}}{\pi_t} = M_t^p - \frac{M_{t-1}^p}{\pi_t} \tag{74}$$

Restricting the initial values leads to the central bank balance sheet condition

$$B_t^{CS} + p_t^L B_t^{CL} = M_t^p \tag{75}$$

In this setting the central bank now controls three instruments. As stated in Chapter 4.4.1, the first two tools are the policy rate and quantitative easing (increasing the size of the balance sheet). In this chapter, with the introduction of long-term bonds, it has been shown that the central bank is able to change the composition of its balance sheet by adjusting the fraction of short-term bonds relative to long-term bonds eligible for reserves. In other words the central bank can now change both κ^s and κ^l.

For the purpose of this model, $0 < \kappa^s \leq 1$ and $0 < \kappa^l \leq 1$. The sum of these can be greater than one, $0 < (\kappa^s + \kappa^l) \leq 2$. Importantly, for the purpose of affecting a change *only in the composition of balance sheet* it must be the case that the sum of κ^s and κ^l remain the same. This is best explained through an example. Consider a case where $\kappa^s = 0.5$ and $\kappa^l = 0.5$. If a change in the composition alone was to be generated, I would have to match the decrease in one value of κ, with an increase in the other. This *sterilising* change can be the following, $\kappa^s = 0.2$ and $\kappa^l = 0.8$. In this new setting, long-term bonds are more eligible as collateral, which makes them relatively more attractive. In addition, it changes the relative bond-holdings of the central bank, which will now accept more long-term debt. This does not, however, change the size of the balance sheet, as the allotment of bonds for liquidity is still the same.

The model from the previous chapter is nested in this model. By allowing $\kappa^l = 0$, it removes the credit easing component from the model, and once again the model is left with only two tools of operation. In summary, this brings about three instruments, interest rate policy, quantitative easing and credit easing. The rest of the chapter investigates the implications of changing the composition of the assets held on the central bank's balance sheet.

To summarise, in this model the central bank has three instruments, namely the following:

1. *Conventional instrument*: The policy rate, R_t^m.
2. *Quantitative easing (size of balance sheet)*: Increase reserves against eligible assets (short-term or long-term bonds) in open market operations by increasing κ^s or κ^l.
3. *Credit easing (composition of balance sheet)*: Changes in the composition of the balance sheet, without affecting size, implemented by a change in κ^l, and met with a sterilising change in κ^s.

Finally, it is also important to point out that the central bank can decide to use these policies in combination, potentially increasing the size and composition of the balance sheet. In the next section, I conduct a partial equilibrium analysis, similar to that of the previous chapter.

6 Equilibrium Conditions

In this section a partial equilibrium analysis is conducted, with specific reference to monetary transmission and endogenous default in the model. Evaluating the equations analytically provides intuition as to the results expected from the model. First, a discussion on the monetary transmission mechanism is provided. There is a vast amount of literature on the transmission of conventional interest rate policy to real activity. Figure **??** from Chapter 2 gives an idea of the agreed upon channels for the transmission of a change in the policy rate, with a brief discussion on these channels provided below. Second, I provide a look at the conditions that need to be met for the collateral constraint on merchant banks to be binding. Third, I provide an analysis on the relationship between default, interest rates and loan activity in the economy.

6.1 Interest Rates

This section highlights the relationship between the policy rate and selected market interest rates. In order to properly frame the discussion, some simplifying assumptions are stipulated. I assume that the non-pecuniary default cost is equal to zero, $\omega_\delta = 0$, repayment rates are constant, $\overline{\delta}$ and $\overline{\psi}$, and the momentary utility for banks is linear. This means that in $\sigma_b = 1$ and $\sigma_l = 1$, which effectively translates to $(\pi_t^b + 1)^{-\sigma_b} \approx 1$ in steady state.

6.1.1 Interest Rate Spreads

First, I consider the relationship between the deposit rate, R^d, and the interbank loan rate, R^l. Intuitively, the presupposition is that the deposit rate should be lower than the interbank loan rate, since the depository institutions would traditionally finance their lending primarily through deposits, making a profit from the difference between rates. To explore this relationship, I unified the first order conditions of the deposit bank. In equilibrium $R^d = R^l \cdot \overline{\delta}$, which means that $R^d < R^l$ since $0 < \overline{\delta} < 1$. In other words, the level of the interbank loan rate is intrinsically linked to the possibility of default, by reflecting a risk premium derived from default. Therefore, in this model, the deposit banks charge a higher rate on interbank loans because they take on a small default risk in issuing these loans. In the case that $\delta = 1$, the transmission from R^d to R^l would be direct, in that $R^d = R^l$.

Furthermore, one can think about this relationship as one between savers, borrowers and intermediaries. In this case, the saver is represented by the household, while the merchant bank takes on the role of the borrower. Intermediaries, represented by the deposit bank, generate a spread between the "interest received by savers and that paid by borrowers" (Cúrdia and Woodford, 2010). Early attempts to fashion such an interest rate spread between borrowers and savers include Sudo et al. (2008), Hülsewig et al. (2009), Cúrdia and Woodford (2010) and Gerali et al. (2010). A more recent attempt that delivers a qualitatively similar mechanism is found in Gertler and Karadi (2011).

Second, I analysed the link between the credit and interbank loan rates by looking at the first-order conditions of the merchant bank. We see that $R^c \cdot \overline{\psi} = R^l \cdot \overline{\delta}$, with the intuition being that a higher repayment rate is associated with a lower interest rate. As calibrated in this model $\overline{\delta} > \overline{\psi}$, which, in turn, means that firms are believed to repay less frequently, which means that the risk premium is relatively higher for credit extended to firms. In other words, merchant banks institute a premium on the financing they receive from the interbank market, making firm loans costlier.

Interestingly, this relation also shows that defaults on firm loans can indirectly impact on deposit banks, through their effect on merchant banks, as also illustrated in Christiano et al. (2014). Another way to think about this, in the context of a disruption to financial intermediation, is that a disturbance in the interbank market will indirectly affect the borrowing conditions for firms, as described in Gertler and Karadi (2013).

Third, the bond rate is related to the interbank loan rate through the following equation, as derived in the first-order conditions of the merchant bank:

$$R^l \cdot \overline{\delta} = R^b(1 + \eta\kappa)$$

In the case where the haircut parameter is positive, $\kappa > 0$, then $R^b < R^l$, which aligns with our intuition, seeing that the interbank loan rate is linked to a risky asset and short-term government bonds (treasuries) are considered completely safe. In addition, the spread between the bond rate and the loan rate is also positive, owing to default risk and the potential for the bonds to be used as liquidity.

Finally, consider the case of how the policy rate is transmitted to the interbank loan rate. Combining the first-order condition on newly issued reserves, money holdings, interbank and firm loans gives the following relation to the interbank rate: $R^m(\eta + 1) = R^l \cdot \overline{\delta} \cdot (\overline{\delta} + 1)^{-1}$. In this setting, with $R^m < R^l$, the central bank directly impacts the interbank loan rate through setting the policy rate. In the literature there are several articles, such as those of Goodfriend (2007), De Fiore and Tristani (2011) and Cúrdia and Woodford (2015), that emphasise the spread between the loan and policy rates in particular.

6.1.2 Monetary Transmission Path

Having completed the transmission path, the relationship between rates reads as $R^m < R^b < R^d < R^l < R^c$. We can use this to determine the transmission from the policy rate to the broader range of interest rates in the economy. For example, lowering the policy rate lowers the cost of credit and potentially increases the supply of loanable funds in the economy. This behaviour is in line with a narrow credit, or bank lending, channel (Bernanke et al., 1999; Cova and Ferrero, 2015). To gain a better understanding of this transmission path, it is useful to look at some of the more popular channels identified in the literature.

6.1.2.1 Transmission Channels Some of the different channels through which a change in the policy rate is thought to affect market rates and loan supply, and thereby the rest of the economy, are presented here[36]. This discussion is useful in that some of the channels are

[36] The effects are summarised in Figure **??** from the discussion of transmission channels in Chapter 2

referenced in the results section of the chapter. The channels, as identified in Mishkin (2001), Kuttner and Mosser (2002) and Ireland (2006) are as follows: (i) interest rate channel, (ii) equity price channel, (iii) balance sheet / broad credit channel, (iv) bank lending / narrow credit channel, and (v) exchange rate channel. Some have also argued for the inclusion of the risk channel[37] of monetary policy, as first discussed in Rajan (2005). I provide a brief discussion on the first four channels.

First, the interest rate channel is the traditional mechanism for the transmission of the policy rate to aggregate demand, as emphasised in conventional Keynesian IS-LM models (Mishkin, 2001)[38]. In this channel an increase in the policy rate would mean an increase in the real interest rate, which translates into an increase in the cost of investment/durable goods. Firms and households invest less as a result, which adversely affects aggregate demand.

Second, the equity price channel contains two separate channels, one which originates from contributions made by Tobin (1969) in his q-theory of investment and the other from the life-cycle hypothesis as presented in Ando and Modigliani (1963). First, consider an increase in money supply, which generally means that households will have more money to spend in stock markets, in turn generating demand for equity and raising its price. In Tobin's q theory, an increase in the price of equity will result in increased investment spending (Mishkin, 2001). Second, the wealth channel as discussed in Ando and Modigliani (1963), specifically looks at changes in household behaviour as a result of asset price movements (Kuttner and Mosser, 2002). Increasing interest rates can have the effect of reducing the value of durable assets (such as bonds with a longer maturity), which decreases the wealth of individuals. A reduction in wealth naturally leads to lower levels of consumption, which means that aggregate demand is impacted.

Third, the broad credit/balance sheet channel was developed by Bernanke and Gertler (1989) and discusses the role that the balance sheet position of key financial institutions play in the transmission of monetary policy. In particular, interest rate movements often bring about changes to the balance sheet, cash flow and net worth of banks and firms (Mishkin, 2001). An increase in the interest rate could, for example, negatively impact cash flow and net worth, which would lead to less collateral being available for loans in the economy. Financial frictions play a key role in this channel, as discussed earlier in this chapter.

Finally, the narrow credit/bank lending channel was first developed by Roosa (1951) and was reformulated by Blinder and Stiglitz (1983) and Bernanke and Blinder (1988). In this channel,

[37] This was briefly discussed in the third chapter. A substantial literature has developed around this topic, with the article by Smets (2014) providing an interesting take on the role of monetary policy in contributing to financial stability.

[38] See the article by (Taylor, 1995) for a good discussion on the empirical validity of the interest rate channel.

an increase in the policy rate is met with an equal decrease in bank reserves, which means that demand deposits will decrease. As a result of the fact that banks are reliant on deposits to generate loans, this also causes loans to decrease. Bank financing is an important part of credit in the economy. With a reduced supply of credit, households and firms will reduce spending, which means lower aggregate demand (Ireland, 2006).

The final two channels are often referred to as the credit channel and are seen more in the light of an amplification device on existing transmission channels, as argued by Bernanke and Gertler (1995). In addition, clear empirical support for certain channels, such as the bank lending channel, have been difficult to come by, with conflicting evidence in the literature.

6.2 Collateral Constraint

Importantly, for the collateral constraint to be binding, the policy rate should be lower than that of interbank liquidity. The central bank needs to supply money at a lower price than banks are willing to pay in the interbank market. The merchant bank must be incentivised to hold reserves (i.e. there is some liquidity premium on the holding of reserves). When we compare these rates we see that this is the case, with

$$\eta = \left[\frac{R^l \left(\frac{\bar{\delta}}{1+\bar{\delta}} \right)}{R^m} \right] - 1 \tag{76}$$

This condition implies that the constraint is binding if the central bank sets the policy rate below the interbank rate. If $\eta = 0$ and the collateral constraint is not binding, changes to the policy rate will not impact the equilibrium allocation of reserves (Hörmann and Schabert, 2015). In this setting merchant banks will use as many eligible assets as possible to get money at the policy rate, up until the interest rate spread (resulting from the liquidity premia) is eliminated, where $R^l \left(\frac{\bar{\delta}}{1+\bar{\delta}} \right) = R^m$.

6.3 Endogenous Default

As before, several simplifying assumptions are made, similar to those established in de Walque and Pierrard (2010), to shed light on the role of endogenous default. I assumed that there is no discounting $\beta = 1$, no inflation $\pi_t = 1$ and utility functions are linear (by setting $\sigma_b = \sigma_l = 1$).

First, we look at the supply side of the credit market. The supply of interbank liquidity, which is provided by the deposit bank, is governed by the following equation,

$$\frac{1}{R^l} = \delta$$

This expression captures the negative relationship between R^l and δ. In other words, a higher repayment rate will result in a lower interest rate on interbank loans . Similarly, with respect to loans to firms,

$$\frac{1}{R^c} = \psi$$

This equation shows the negative relationship between the repayment and credit rates. Second, we explore the demand side of the credit market. Combining the merchant bank's first-order conditions for interbank loans and the repayment rate, we find the following steady-state relationship:

$$\frac{1}{R^l} = 1 - \frac{d_\delta(1-\delta)}{L^l}$$

This indicates a negative relationship between the quantity of interbank loans and the interbank rate, while a positive relationship is observed with respect to the default rate, $(1-\delta)$, and interbank rate. Similarly for firms, I combined the first-order condition with respect to firm loans and default,

$$\frac{1}{R^c} = 1 - \frac{d_\psi(1-\psi)}{L^b}$$

This has similar implications for the firm. For example, an increase in the credit rate will lead to a lower quantity of loans to firms. de Walque et al. (2010) refer to this as "negatively sloped credit demand". In addition, an increase in the default rate, $(1-\psi)$, is associated with a lower quantity of loans, indicating that increases in liquidity to the firms will generally lead to lower default.

7 Equilibrium Conditions

In this section I provide an analytical evaluation of the monetary transmission mechanism. In order to evaluate properly the equilibrium conditions some hermeneutic simplifications are stipulated. Equilibrium conditions for endogenous default are quite similar to those of Chapter 6.1. Changes in long-term bond rates affect default through their relationship with credit and interbank rates. This relationship is discussed in the following section.

7.1 Monetary Transmission

As discussed in the previous chapter, interest rate spreads are at the heart of the transmission mechanism in models that include several markets and differing asset classes. These spreads are indicative of the health of the financial system at large, as experienced during the recent crisis[39]. In order to evaluate the conditions effectively, I reduced complexity by adopting the same assumptions as in Chapter 6.1. I also added the assumption that inflation is zero, $\pi = 1$.

In our current setting, the only new addition to the pool of interest rates is that of the long-term bond rate. The new monetary transmission path relies on the relationship between the short- and long-term bond rates, which is presented as

$$\frac{1 - \kappa^s \eta R^b}{R^b} = \frac{1 - \kappa^l \eta R^L}{R^L - \Phi}$$

This relationship indicates the importance of Φ in creating a wedge between the short- and long-term rates. If $\Phi = 0$, then the long-term bonds have the same maturity as their short-term counterparts. The larger the value of Φ, the greater the interest rate spread (i.e. a higher long-term rate is achieved). Extending the maturity increases the overall riskiness and, thereby, the yield of the underlying asset.

8 Calibration

Calibration of the household, firm and government largely follows the work of Smets and Wouters (2003, 2007), while monetary policy and the banking sector is calibrated to reflect the values in line with de Walque and Pierrard (2010). Model parameters are partitioned into sets or groups, similar to those of Christiano et al. (2014). In this tradition, the first group contains parameters that are set *a priori*. Within this set, calibrated parameters are summarised in Table 1, while imposed steady states and steady-state ratios are presented in Table 2. The second set consists of implied steady state values, such as those on search costs and default disutility, which are provided in the appendices. The discussion in this section highlights the motivation for the first set of parameters.

[39] See the article by Cúrdia and Woodford (2011) for a discussion on the importance of interest rate spreads. Also look at the brief discussion in Chapter 2.1.2.

8.1 Real Sector

The calibration of the real sector arises predominantly from widely accepted parameter estimates found in the New-Keynesian literature. As seen in Table 1, habit formation $h = 0.57$, the coefficient of relative risk aversion $\sigma^c = 1.35$ and inverse Frisch elasticity of labour supply $\sigma^n = 2.4$ are similar to those found in Smets and Wouters (2003). The elasticity of substitution with respect to labour η and goods ϵ varieties are both set to 3. In terms of price/wage setting, the wage indexation and Calvo parameters are set to $\tau^w = 0.62$ and $\theta^w = 0.2$, respectively. Output is normalised to 1. Employment/labour hours is normalised to 0.33. The production function takes on a Cobb-Douglas form, with $\alpha = 0.3$, implying that steady-state labour share of total output is 70%. The depreciation rate φ is calibrated to 0.03, with annual depreciation on capital equal to 12%. The consumption steady-state ratio is assumed to be 0.42, while the firm profit steady state ratio is 0.1[40]. Calibration implies that the capital to output ratio is $K/Y \approx 8$ and the loan to output ratio is $L^b/Y \approx 0.2$, which is similar to those of Smets and Wouters (2003) and de Walque et al. (2010). The government spending and taxation ratio to total output is 0.185 and 0.187, respectively. There are three parameters associated with adjustment costs in this model; these are defined in terms of price $\varrho = 120$, wage $\gamma_p = 0.47$ and investment $\theta = 6.77$.

8.2 Banking Sector

Calibration in this section largely follows the work of de Walque and Pierrard (2010). Several interest rates are quoted in this paper. The quarterly real deposit rate is 0.66%, and the quarterly real policy rate is 0.55%. The discount factor is calculated as $\beta = \pi_{ss}/R^d_{ss}$, which I fixed to be the discount factor for all agents in the model[41]. Following data from Castrén et al. (2010), the quarterly probability of default for banks is 0.5%, whereas the firm default rate in steady state is 2.5%. Deposit and merchant bank profits are quite small related to total output, namely $\pi^b/Y = \pi^l/Y = 0.0001$. I also imposed a ratio of interbank loans to firms loans in steady state as in de Walque and Pierrard (2010), with $L^l/L^b = 0.8$ [42].

[40] The consumption ratio is normally larger, refer to Smets and Wouters (2003), but in our case firm and bank profits are not distributed to the household

[41] Relaxing this assumption allows different discount rates for the households, firms and banks.

[42] Changes to these ratios do not significantly impact results, but they do influence some of the implied parameter values. Ratios were chosen close to the ones cited in de Walque et al. (2010) but are not exactly the same, in order to retain a positive value for the endogenous disutility and search cost parameters.

9 Calibration

Changes in terms of calibration occur primarily in the banking and central bank sectors. This discussion highlights only the differences between the model in this chapter and that of the previous chapter, with a summary of the newly calibrated parameters and imposed steady-state values provided in Table 1 and Table 2 in Appendix C.

The long-term rate was imposed and set at $R^L_{ss} = 1.075$, which is close to the calibrated value presented in Niestroj et al. (2013) for the Euro area. This value maintains the hierarchy of interest rates, as discussed in the monetary transmission section. The average duration of the long-term bond was set at 5.5 years[43], which means that the coupon rate of the perpetuity can be calculated as $\Phi = (5.5 \cdot R^L_{ss} - R^L_{ss})/5.5$. Implied steady-state values are further discussed in Appendix C.

10 Model Dynamics

In this section, I looked at the impact on financial stability originating from shocks to (i) the haircut mechanism, κ_t, and (ii) the nominal short-term policy rate with selected levels for κ. First, I looked at the financial market impact of implementing a balance sheet expansion, through an increase in the value of κ_t. This first scenario is similar to the analysis of de Walque et al. (2010), Dib (2010a), Hilberg and Hollmayr (2011) and Goodhart et al. (2011), who employ expansionary monetary policy through an increase in the monetary base. The model differs, primarily, in the way that the balance sheet expansion is implemented. In this model, the collateralised lending mechanism provides an endogenous provision of liquidity, as in Schabert (2015), rather than a pure injection.

Second, I conducted an experiment that simulates a scenario, similar to the exit strategy recently performed by the Fed, whereby the nominal short-run policy rate is increased, with varying degrees of change to the size of the central bank balance sheet, reflected by different κ values. The similarity of this scenario to that of an exit strategy lies in the fact that the Fed was able to increase the federal funds rate without significantly impacting the size of the balance sheet. In this model, values of κ can be set so that reserves move independently from the policy rate, in order to achieve a similar result. In this scenario the point of departure does not have to be from the ZLB. I implemented this Contractionary interest rate policy by the applying a shock to the Taylor rule (58).

[43]This was modelled on the average duration of 7-year US Treasury bills, but the value could be changed, with a duration of 7.5 years as an alternative in the case of a 10 year Treasury (Chen et al., 2012).

It is worth mentioning that model estimation was not attempted. The contribution of the paper is in its theoretical construction, with calibration being sufficient for the purpose of exposition. Clear data adherence in the model is a topic for future research. The analysis starts with the expansion of the central bank balance sheet.

10.1 Balance Sheet Expansion

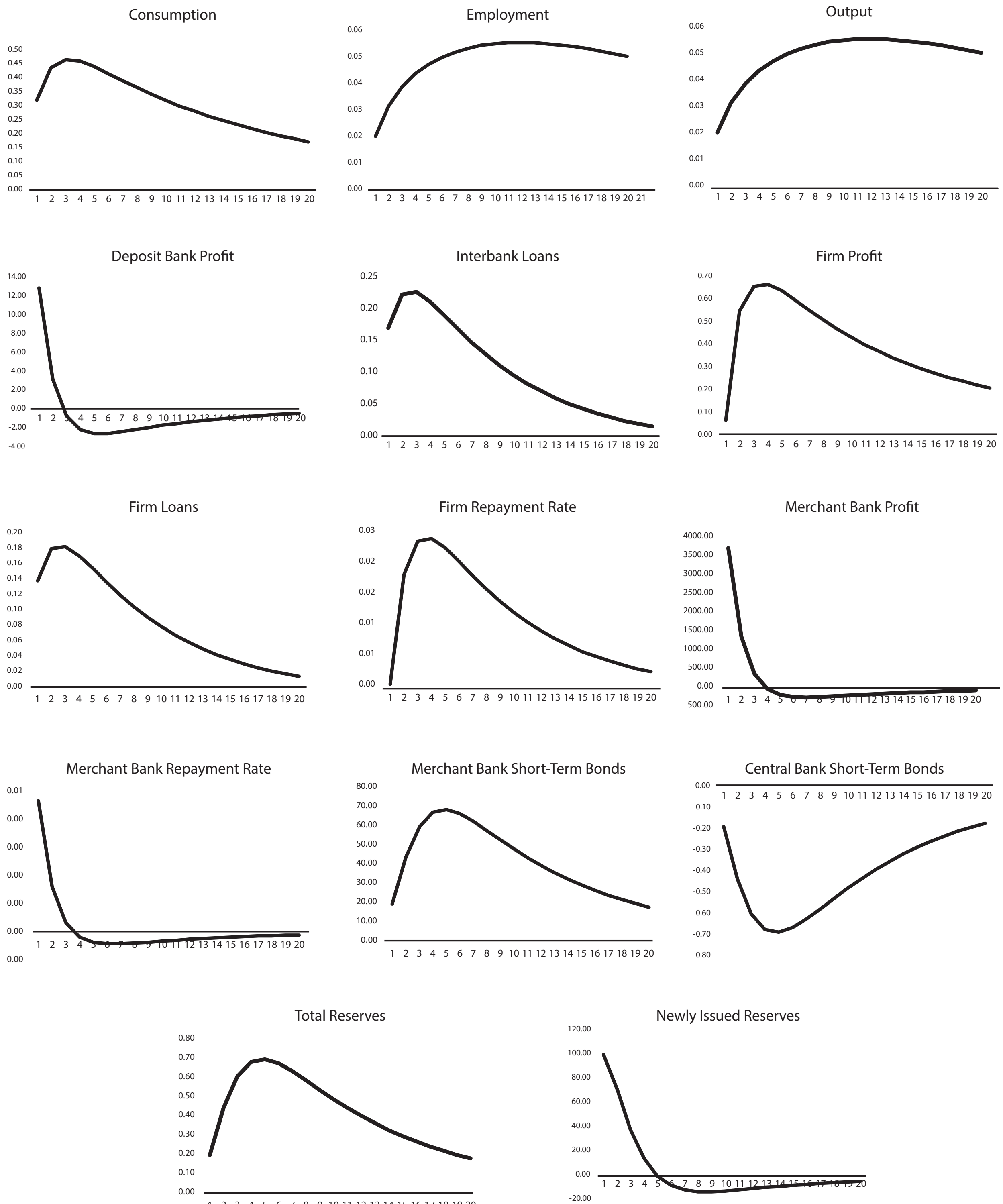
One can think about this expansion as an increase in the availability of reserves. In other words, it represents a 'pure' form of QE, such as that stipulated in Bernanke and Reinhart (2004), Reis (2009), Woodford (2012) and Christensen and Krogstrup (2016). First, I describe the real sector impact from the balance sheet expansion. Second, I look at the effect on the financial sector, which is of the greatest importance for this discussion, as I want to determine the impact on financial stability from balance sheet expansion. The results from this section should be quite similar to those of a decrease in the policy rate, as the level of reserves and policy rate are inextricably linked through the collateralised borrowing mechanism.

10.1.1 Clarification: Variable (κ_t) vs Parameter (κ)

Clarification on one possible point of confusion is required before I continue with a discussion on the model dynamics. In this section I consider an exogenous shock process for the collateral requirement, κ_t, which follows an autoregressive process. With the balance sheet expansion, the law of motion for this process is $\kappa_t = \rho_\kappa(\kappa_{t-1}) + \xi_t^\kappa$, where ξ_t^κ is an exogenous i.i.d shock. However, in the next section, I consider a fully adjustable haircut parameter κ, in the form of a scalar. For the scenarios presented in the next section, where a contractionary interest rate policy is considered, I believe it to be more instructive to simply set parameter values that remain constant over time[44], which allows our analysis effectively to be comparable to the study of Goodhart et al. (2011), where the value of κ is implicitly set to one. In terms of an approximation to reality, haircuts are not generally time-varying as they are officially fixed by central banks (Falagiarda and Saia, 2013). This means that fixing κ to certain value over time is considered a better representation of actual implementation.

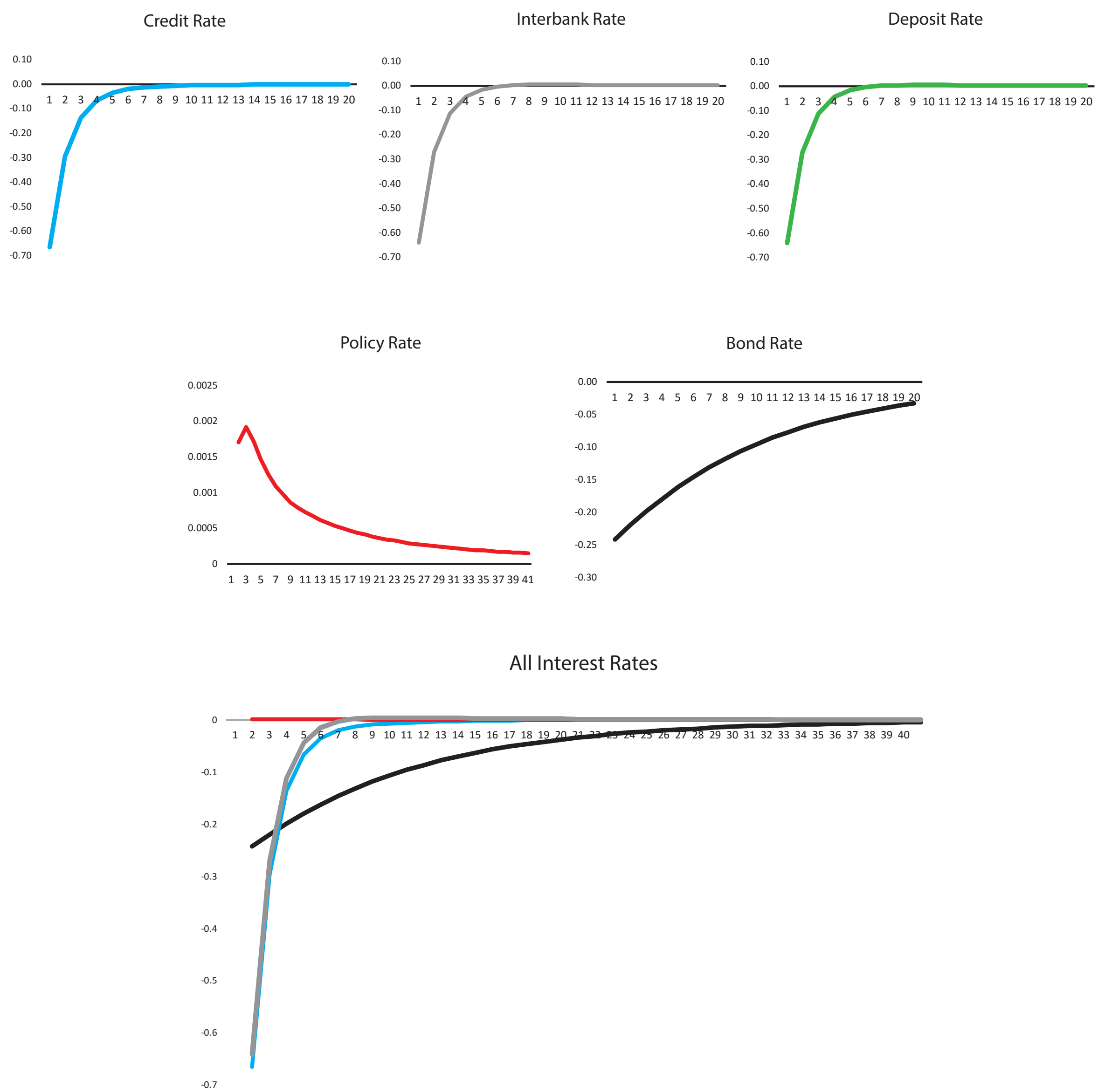
[44]Technically this is similar to setting the autoregressive parameter on the haircut process to one, which also delivers a constant value over time.

Figure 1: Balance sheet expansion: Real and financial sector



Note: Y-axis values represent percentage deviation movement from steady state, resulting from 100% shock to collateral requirement

Figure 2: Balance sheet expansion: Interest rates



Note: Y-axis values represent percentage point deviation from steady state from 100% shock to collateral requirement

10.1.2 Real Sector

Illustrating the effect of a shock to κ_t entails setting the starting value of κ_t to below one[45]. In my experiment the value is fixed at 0.5 initially, which implies that agents are liquidity constrained, with only a fraction of securities available as collateral[46]. With the policy adjustment, the value for κ_t increases so that the LTV ratio is close to, but not exceeding, 100%. An increase in κ_t means that more bonds are becoming liquid (Hörmann and Schabert, 2015). The expansion of the central bank balance sheet (often referred to as QE) is reflected in a positive innovation to κ_t, which affects the pertinent real variables, showing the expected sign and trajectory. Real consumption, expenditure and employment all increased with the expected hump shape, a characteristic of the Smets and Wouters (2003, 2007) framework. Real sector results from the next section, with contractionary monetary policy through an increase in the policy rate, almost identically mirror the results from this increase in κ_t.

10.1.3 Financial Sector

While real sector results are important to determine whether the model provides an accurate description of the broader macroeconomy, I am primarily interested in the banking sector behaviour. The central hypothesis of this chapter pivots on the response of financial institutions to variation in the collateralised borrowing constraint. Therefore, I focus the greatest part of the discussion on the relevant actions of the agents involved in financial intermediation, specifically looking at the behaviour of the firm, deposit bank, merchant bank and central bank.

10.1.3.1 Deposit Bank First, the reduction in the price of interbank funds (i.e. the interbank rate), as a result of the increase in liquidity, translates to increased interbank activity. This finding is shared in the work of Dib (2010b), which points to a possible increase in interbank borrowing following a negative shock to the haircut. My results are further corroborated by the findings of Hilberg and Hollmayr (2011). In their study lowering the haircut results in a decrease in the interbank market rate as well as an increase in interbank lending activity[47]. Balance sheet expansion and lower interbank rates mean that fewer merchant banks are likely to default on interbank loans. This result agrees with that of Goodhart et al. (2011), who state

[45]With a value of $\kappa = 1$ representing complete relaxation of the constraint on the amount of collateral eligible for the purpose of transformation to a more liquid asset. Seeing as a positive innovation is applied to κ, with $0 < \kappa \leq 1$, the initial value needs to be below one, otherwise the constraint is violated.

[46]This starting value is in fact close to the haircut imposed on some private sector investors in the recent financial crisis.

[47]Banks are structured in a slightly different way in their model, with the deposit bank the one with direct access to the central bank's reserves

that the expectation of a higher level of credit in the economy causes the probability of default to decline. The profitability of the deposit bank increases because the marginal benefit from extending loans exceeds the cost.

10.1.3.2 Firm Second, the expansionary balance sheet policy increases the firm's profit, due to reduced capital costs and an increase in cheap liquidity available from merchant banks[48]. The increase in liquidity originating from the shock to the collateral requirement causes a decrease across a wide range of interest rates in the economy, with the credit rate exhibiting a sharp contraction. With borrowing conditions improved, one observes an increase in loans extended to firms. Easier credit market conditions manifests in an increase in the repayment rate on loans (i.e. the default rate decreases); effectively improving financial stability with respect to the firm. Improved balance sheet health is one of the primary goals of liquidity extension on the part of the central bank (Bernanke, 2012). In the wake of the financial crisis several central banks established liquidity facilities and enacted quantitative easing programs in order to generate firm borrowing. Results presented here provide confirmation that liquidity injections should be, in principle, able to accomplish this.

10.1.3.3 Merchant Bank Third, reserves held on the balance sheet of the central bank increases as a result of the shock. Merchant banks increase their demand for reserves, as short-term bonds can now easily be traded for liquidity. In fact, they increase their stock of short-term bonds - which serve as collateral for reserves - to finance their spending, which drives down the price on these bonds. This result is supported by the work of Kandrac and Schlusche (2015), who show that "lending growth accelerates in response to increases in reserves". It is even argued as far back as Friedman and Schwartz (1963) that the creation of reserves leads banks to hold more than the sufficient level, which would then translate into an increase in investments and loans extended in the economy. This result is also consistent with the empirical evidence, as the recent study by Boeckx et al. (2016) shows that balance sheet expansion on the part of the ECB significantly increased bank lending. In addition, the credit supply to firms is increasing due to a positive spread between the credit and interbank rates. The central bank's role as an intermediary is clearly felt in this economy, with borrowing conditions improving in all markets when the collateral requirement is relaxed.

10.1.3.4 Central Bank Finally, the increase in the reserves held on the balance sheet of the central bank is met with an increase in the policy rate. While this movement might seem counterintuitive, the increase in the policy rate is also observed in the work of Niestroj et al.

[48]In this model the merchant bank is the firm's only source of funding for investment

(2013). It is believed that this is the result of the way in which the feedback mechanism from the Taylor rule is structured. The increase in output resulting from the increase in the size of the balance sheet means that the policy rate increases to counter the movement. However, the net effect of the balance sheet expansion is positive in this case, with movement in the short-run nominal policy rate not being able to counter the effect of the expansion, as seen by the fact that all other interest rates in the economy are declining (in contrast to the policy rate). In the next section, I look at the impact on the economy from implementing a contractionary interest rate policy.

10.2 Contractionary Interest Rate Policy

In this section, I consider the response of the economy when the short-term nominal policy rate has been increased, known as contractionary monetary policy. I introduced multiple scenarios with a choice over different values for an exogenously controlled haircut parameter[49]. Four values for the haircut at different intervals, 1, 0.7, 0.3 and 0.05 were chosen for the analysis. These values represent a 100%, 70%, 30% and 5% LTV ratio, respectively. The value of $\kappa = 1$ was chosen to reflect the situation in which there is no collateralised lending restriction: all short-term bonds are eligible. This formed my baseline specification.

For the second value, the literature provides an LTV ratio of $\kappa = 0.7$ as a realistic representation from data for the Euro area on mortgages offered as collateral, such as quoted in Gerali et al. (2010). Although mortgage securities are not directly relatable to short-term bonds, one can think of this as realistic to some asset classes with lower eligibility as collateral. The true value for short-term bonds would be closer to $\kappa = 0.9$, but this would not allow the exaggeration of the effect required, as it is too close to the baseline model where $\kappa = 1$. Finally, a value of $\kappa = 0.3$ was chosen to reflect a deepened but not entirely unrealistic haircut[50], representing a scenario of low interest rate sensitivity[51]. Finally, the value of $\kappa = 0.05$ is an effective decoupling of the interest rate and reserves.

10.2.1 Real Sector

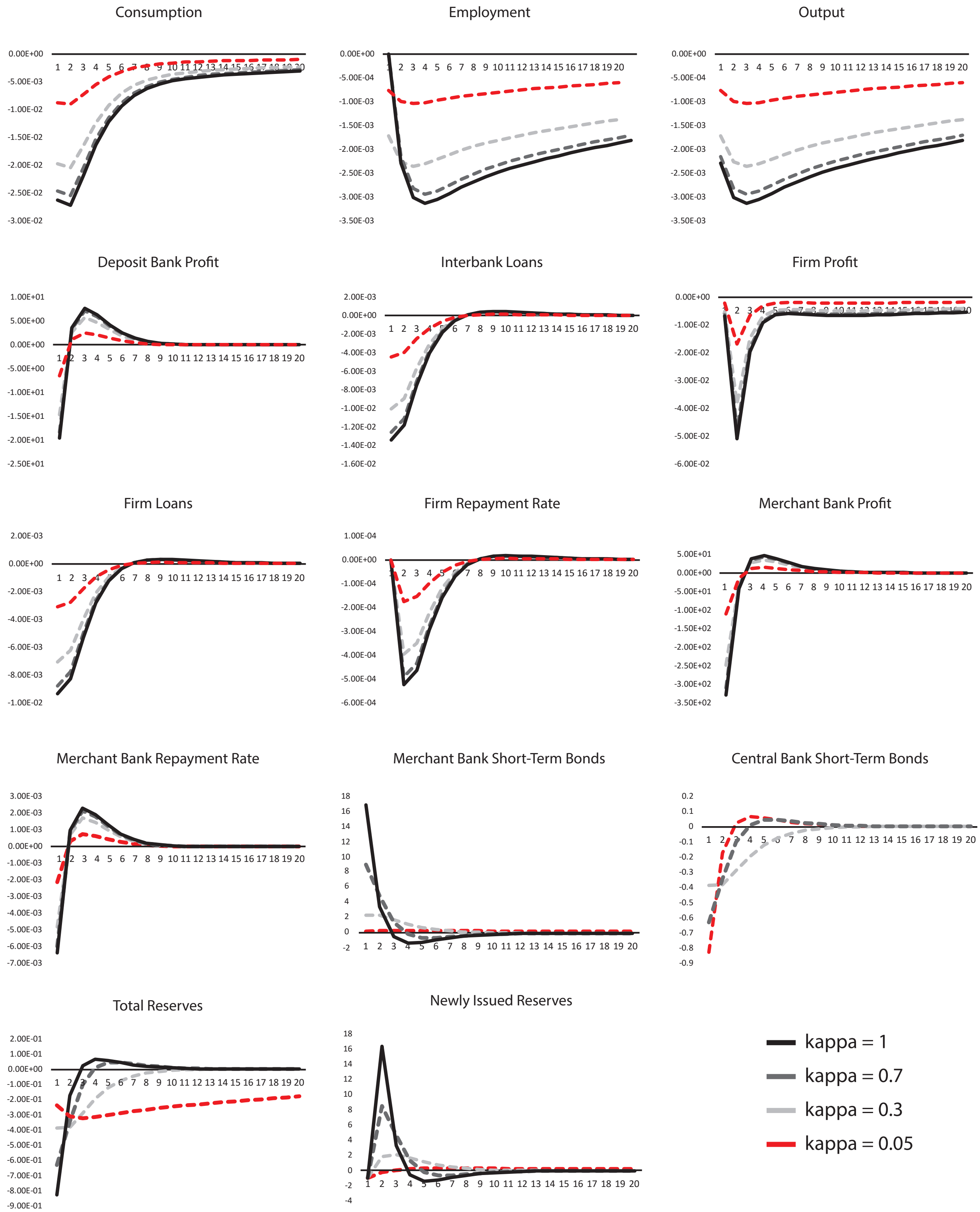
The first few panels show the traditional hump-shaped responses to a monetary policy tightening, as in Smets and Wouters (2003, 2007). The results are similar for all iterations of κ. In all the

[49] As opposed to the shock form the previous section.

[50] In fact, the ECB under their liquidity provision programs extended loans to some financial institutions where the haircut was close to 30%; normally on bottom tier asset classes (Gerali et al., 2010)

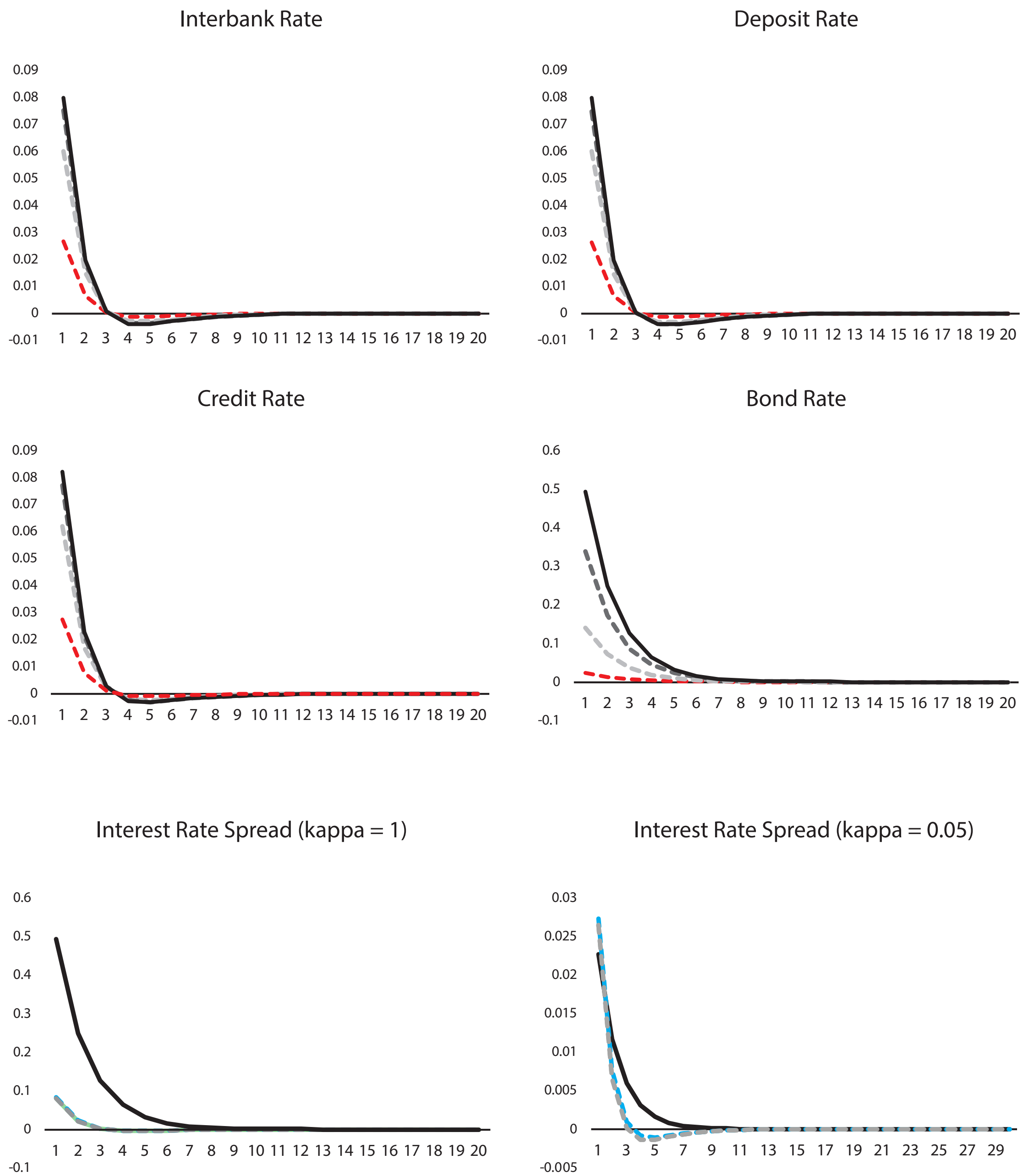
[51] A true decoupling would require a value of $\kappa = 0$, however, setting the value for κ too low causes indeterminacy in the model. The value of 0.05 was chosen because lower levels deliver similar results.

Figure 3: Contractionary interest rate policy: Real and financial sector



Note: Y-axis values represent percentage deviation from steady state, resulting from 1 percentage point shock to the policy rate.

Figure 4: Contractionary interest rate policy: Interest rates



Note: Y-axis values represent percentage point deviation from steady state, resulting from 1 percentage point shock to the policy rate.

stated scenarios a drop in output, inflation, employment and investment is observed, as expected in the case of a policy rate increase. All these variables fall and then gradually converge to their steady states.

In this scenario, it appears that with lower the value of κ, the more the effect of the interest rate increase is dampened. For example, the realised impulse response values for consumption in Figure 3 are progressively higher with successive reductions in the haircut parameter. Intuitively, this is the result of a relatively more accommodative central bank balance sheet position, which is captured by the relatively lower values of κ.

10.2.2 Financial Sector

Real sector results from my model largely corroborate the empirical findings from comparable dynamic general equilibrium models. With the discussion on real sector effects exhausted in other studies, it affords me the opportunity to shift the focus of the analysis to the financial sector. The discussion follows a simple template. First, the impulse responses associated with each financial institution are evaluated on the basis of the benchmark case, with no constraint on central bank lending (i.e. $\kappa = 1$). Second, the baseline is compared with the scenarios in which the haircut constraint is imposed (i.e. $\kappa = 0.7$, $\kappa = 0.3$ and $\kappa = 0.05$). Another way to frame this second scenario is to consider the usage of interest rate policy while balance sheet policy is bounded, in the same vein as Harrison (2012).

10.2.3 Clarification: Decoupling Principle

In order to understand the results better, one important clarification is necessary, as there are potentially conflicting effects occurring. Usually in open market operations conducted to affect a negative impact on the short-term interest rate, the central bank would drain reserves from the economy by selling short-term securities to (merchant) commercial banks. In the model presented the initial decrease in the interest rate, with $\kappa = 1$, is met with a relatively large decrease in the amount of reserves. This is expected, as the value of κ, along one dimension, represents a sensitivity of reserves to the movement in the interest rate. Lower values of κ result in lower initial decreases of reserves held by merchant banks (i.e. a higher relative level of reserves), indicating a lower responsiveness of reserves to changes in the interest rate. This relates to the discussion on the decoupling principle in the previous chapter.

However, on the other hand, merchant banks in the scenario where $\kappa = 1$ have accumulated more short-term bonds to trade for reserves (as they yield higher returns with an increase in the policy rate) and also have the means to convert these bonds to reserves, over the longer-run. By this I mean that all their short-term bonds can be offered as collateral in obtaining liquidity: there is no collateral restriction. In comparison, while the initial reduction in reserves is relatively low for lower values of κ, the exchange of short-term bonds for reserves operates more slowly with lower values of κ. Therefore, in the case where $\kappa = 1$, for example, the initial reduction in reserves is sharp due to the interest rate increase. However, in this instance the merchant bank has greater means to obtain (now higher yielding) reserves in exchange for short-term bonds, allowing a potentially quicker recovery.

10.2.3.1 Deposit Bank ($\kappa = 1$) First, in the case of the deposit bank - which provides liquidity in the interbank market - an increase in the policy rate gets partially transmitted through to the interbank interest rate, creating a positive spread between the policy and interbank rate. As indicated in the partial equilibrium analysis, this spread is generated primarily by the size of the merchant bank default parameter, δ.

Given the higher price on interbank market investment, the demand for interbank loans decreases (Goodhart et al., 2011). Deposit bank profits are negatively affected by the contractionary policy at first, but then steadily increase over the next few quarters to surpass its steady-state value. Profits increase, with interbank loans starting to accumulate and the interbank rate returning to its pre-shock levels. While this result might seem counter-intuitive, there is evidence to support it in the literature[52]. Giri (2014) states that the policy rate increase appears to benefit the net creditor/surplus bank in the economy. In fact, Goodhart et al. (2011) refer to the deposit bank as the net lender in their paper. The intuition is that net creditors are eventually able to take advantage of the fact that interbank lending now occurs at a higher price.

10.2.3.2 Deposit Bank ($\kappa = 0.7$; $\kappa = 0.3$; $\kappa = 0.05$) Reducing the value of κ induces a relative increase in central bank reserves in the short-run, as the result of reduced interest rate sensitivity of reserves. To be clear, this initial expansion of the central bank balance sheet is a relative one, as it is in comparison to the case where reserves are more reactive to movements in the interest rate, namely $\kappa = 1$. The most striking impact from this relative increase in reserves is the relatively higher level of loans extended on the interbank market, which is most readily observed with $\kappa = 0.05$. Interestingly, it appears that the price of interbank lending is a function of the amount of loans, with the pass-through of the policy rate on the interbank rate being relatively stronger in the increased liquidity environment.

Deposit bank profit decreases less sharply to the contractionary interest rate shock with lower

[52]In the bank lending channel, it is possible for a supply-driven bank to be profitable with an increase in the interest rate (Kuttner and Mosser, 2002).

values of κ, dampening the negative effect of the policy stance. However, profit recovery is faster with higher values of κ, leaving the deposit bank with a marginally higher level of profit in the longer-run. This appears to be linked to the movement of the interbank rate, showing the interest rate sensitivity of profits.

10.2.3.3 Firm ($\kappa = 1$) Second, I look to the response of the firm. As expected, firm loans decrease due to the increase in the loan rate imposed by the monetary policy shock. Firm profits decrease, which confirms my intuition. In addition, firm repayment decreases, which translates into an increase in the probability of firm default. There appears to be a positive correlation between loans to the firm and the repayment rate, as highlighted by the partial equilibrium analysis. In other words, the lower the number of loans extended to firms, the higher the probability of default. This could indicate that in times when firms are credit constrained it becomes more difficult to repay their debt. The number of loans to firms decreases with the increase in the real credit rate and decline in the value of firms' capital.

10.2.3.4 Firm ($\kappa = 0.7$; $\kappa = 0.3$; $\kappa = 0.05$) When the central bank broadens its role as intermediary, through a decrease in κ, there is a relative decrease in the credit rate. Also, the quantity of firm loans continues to increase in line with the initial expansion in the central bank's balance sheet. The increase in firm loans contains elements of both demand and supply. On the one hand, firm demand could be heightened because of easier borrowing conditions. However, an increase in the liquidity available to the merchant bank is also a factor in the increased number of loans extended, indicating that the determination of loan quantity is not purely based on movements in the credit rate.

10.2.3.5 Merchant Bank ($\kappa = 1$) Third, I considered the case of the merchant bank. Before discussing the result, it is important, once again, to highlight the context for the scenario at hand. Looking at the collateralised lending equation (44), one observes that central bank intermediation directly affects the merchant bank in the model, as it receives reserves from the central bank in return for government bonds proportional to the haircut parameter. Another important interpretation of the equation, in the case where $\kappa = 1$, is that the relationship between the policy rate and newly issued reserves is one-to-one. In other words, interest rate sensitivity is defined by this parameter. In this case, with $\kappa = 1$, an increase in the policy rate should result in a commensurately negative response in reserves, i.e. as used in Goodhart et al. (2011). However, the results from Chapter 5 indicate that this sensitivity parameter might take on a range of values, depending on the monetary policy regime in question[53].

As expected, merchant banks experience a negative effect on their profit from the contractionary policy, due to the increased cost in external financing. The number of bonds in the merchant bank portfolio increases initially, for several plausible reasons. First, open market operations by the central bank place short-term securities with the merchant bank in return for reserves. It is difficult to see from the figure, but newly issued reserves are decreasing initially. This decrease is overshadowed by the accompanying increase in demand for reserves, with merchant banks aggressively pursuing liquidity as a result of the contractionary policy imposed.

Second, an increase in short-term bonds - as they are convertible to reserves in future - reflects the substitution away from the relatively more expensive interbank market as source of liquidity. This finding is corroborated by Giri (2014), who finds that these banks move away from the interbank market toward the purchases of government bonds in the case of monetary tightening. After the initial increase, the merchant bank short-term bond supply decreases rapidly, as it is exchanged for reserves. This then, leads to sharp, but temporary, increase in total reserves. With the increase in the policy rate, the merchant banks want to deposit more reserves at the central bank, a result that is shared by Hilberg and Hollmayr (2011). Finally, the increasing cost of borrowing, as a result of the monetary policy shock, increases default rates for the merchant bank.

10.2.3.6 Merchant Bank ($\kappa = 0.7$; $\kappa = 0.3$; $\kappa = 0.05$) Decreasing the value of κ decreases the level of bonds held by the merchant bank, relative to the benchmark (i.e. $\kappa = 1$). Importantly, this increased collateral requirement reduces the amount of reserves that can be acquired by the merchant bank (Niestroj et al., 2013). In other words, after a few periods, the tightening of the collateral constraint (with lower values of κ) results in a relatively lower level of reserves being available to the merchant bank.

Lowering the value of κ results in an increase in the repayment rate on interbank loans. In addition, an increase in the repayment rate from loans issued to firms is observed. Merchant banks are affected by lower borrowing costs and, therefore, lend more to their customers, i.e. the firms (Goodhart et al., 2011). With lower values of κ, the liquidity position of the merchant bank improves to the extent that it can better service firm loan demand. Both firm and merchant bank profitability increase as a result of the liquidity improvement. In summary, banks anticipate that the increase in liquidity/credit would mean an overall higher level of

[53] The results from Chapter 5 also reveal that there is rarely occasion to believe that the value for κ would strictly be equal to one. The only instance where this is truly possible is in a reserve regime, but even in this regime it is likely that the sensitivity is lower than one. In other words, the elasticity of demand for reserves might be lower on the money demand curve.

credit extension. The probability of default for both banks and firms will decrease, because of the liquidity position and the increase in profitability associated with this constraint (Goodhart et al., 2011).

10.2.3.7 Central Bank Finally, the central bank increases the policy rate, which results in an initial decrease in both total and newly issued reserves[54]. This negative relationship indicates that the decoupling is not complete, with lower levels of κ generating a greater decoupling of the quantity and price of reserves. In other words, for lower values of κ, we see that the reserve sensitivity to interest rate changes declines, with a subdued effect on the reserves for $\kappa = 0.3$. Selecting a parameter value at the lower end of the spectrum produces an interest rate increase with little impact on the number of reserves.

10.2.3.8 Interest Rates The spread between interest rates changes significantly with different values of κ. Decreasing the value of the haircut parameter narrows the spread between the bond rate and other market rates. This decreased spread could be indicative of improvement in the market conditions. As discussed in Cúrdia and Woodford (2010), one would expect tighter financial conditions to be met with an increase in interest rate spreads. In addition, higher values of κ represent an increased ability for the policy rate to affect market rates. The pass-through of the policy rate increase is experienced most intensely in the movement of the bond rate.

10.2.3.9 Exit Strategy In order to generate an exit from the ZLB, the value of κ would have to be as close to zero as possible. The current model would, however, have to be extended along several dimensions to capture the full effects of such a strategy. The work from this section points to the possibility that there would be a short-run gain from severing ties with the balance sheet in the case of an interest rate increase. This is because reserves do not decrease with a rise in the policy rate under decoupling. In the longer-run, in this model, money supply might be higher with a decoupling, even if banks are constrained by their inability to exchange bonds for reserves. The decoupling effect dominates in this model, with improved financial stability and real economic performance associated with lower values of κ. However, this analysis does not capture the full complexity of the strategy, which is why it is deferred to future research.

This result does, however, raise an important question. In the case where central banks increase the interest rate, with the balance sheet completely decoupled (such as a floor regime) it would mean that banks hold a higher than normal level of reserves. This might act to partially

[54]Once again the initial decline in total reserves is difficult to see on the graph.

counteract some of the influence of the interest rate increase. Policymakers need to weigh the cost and benefits of implementing these policies in the light of this result.

11 Model Dynamics

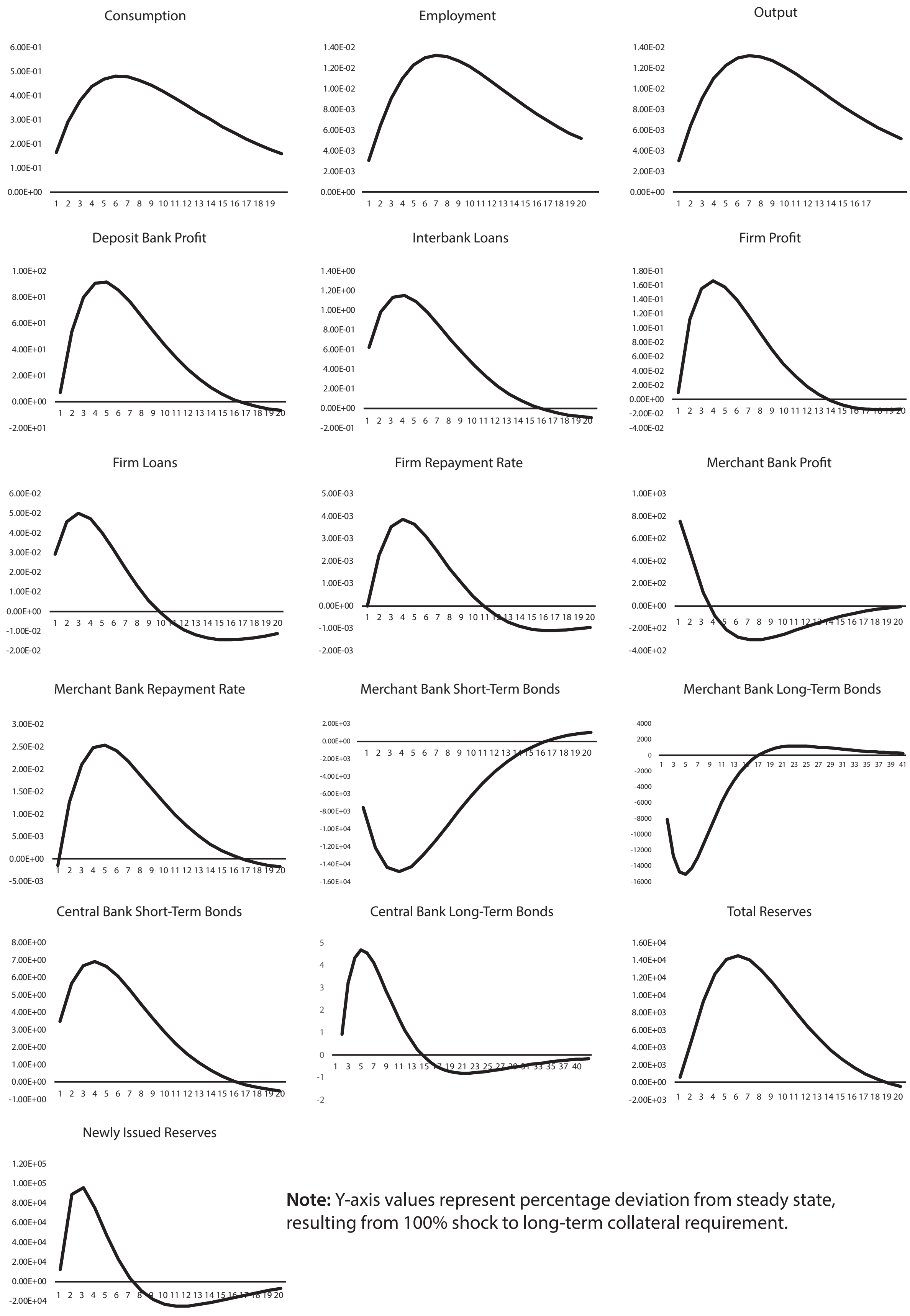
Following the structure of Chapter **??**, I analyse the impact of several different shocks and associated scenarios. First, a shock is applied to κ_t^l, which represents the proportion of long-term bonds eligible as collateral in open market operations. This shock produces results similar to those of large-scale asset purchases, by inducing demand for a long-term bonds. I compare this result with (i) a shock to the long-term bond growth rate, and (ii) the balance sheet expansion result from Chapter 6. Second, I introduced contractionary monetary policy with different combinations of κ^s and κ^l, to affect changes to the composition of the balance sheet. I am particularly interested in the impact that varying these parameters could have on default rates, the extension of loans to firms, and interbank trading. In addition, long-term yields and several interest rate spreads were examined to determine whether the policy performed as expected.

11.1 Large-scale Asset Purchases

Large-scale asset purchases in this section of the model entail both an expansion of, and a change in the composition of assets of the balance sheet of the central bank. In other words, it will be a combination of quasi-debt management and reserve-supply policy from the typology of Borio and Disyatat (2010), similar to actual LSAPs conducted during the crisis. The way I approached this was to apply a shock to κ_t^l, which represents the eligibility of long-term bonds as collateral, while keeping κ^s fixed. An increase in κ_t^l is intended to simulate the effect of increasing the quantity demanded of long-term bonds (relative to short-bonds) for the merchant bank. Modelling the LSAPs in this way aligns more with the LTROs implemented by the ECB. In this scenario assets are not directly purchased by the central bank, rather, there is a change made in the bonds that are eligible for collateral (Gertler and Karadi, 2013).

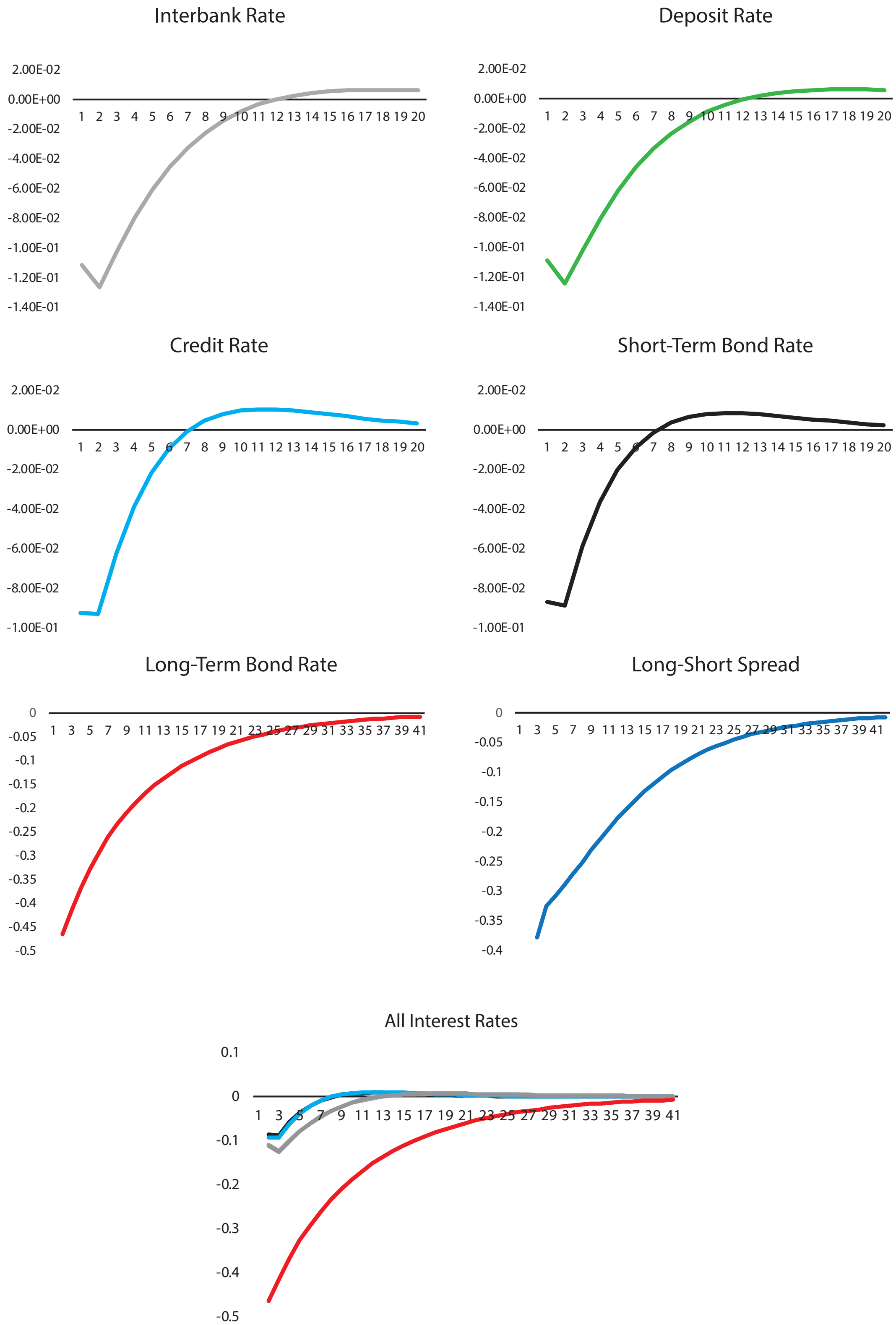
In order to do this, κ_t^l is presented as a time-varying variable following an autoregressive process. Specifically, the haircut on long-term bonds, as a variable, follows an AR(1) process, such that $\kappa_t^l = \rho_l \kappa_{t-1}^l + \xi_t^l$, with $\rho_l \in (0, 1)$ and $\xi_{l,t}$ an i.i.d. exogenous shock, while keeping the haircut on short-term bonds, κ^s, constant over time (i.e. takes on a scalar value). Both the haircut mechanisms on short- and long-term bonds will have the same initial value, with a shock then imposed on the long-term bond haircut value. This expansionary shock should simulate the general function of LSAP policies.

Figure 5: Large-scale asset purchases: Real and financial sector



Note: Y-axis values represent percentage deviation from steady state, resulting from 100% shock to long-term collateral requirement.

Figure 6: Large-scale asset purchases: Interest rates



Note: Y-axis values represent percentage point deviation from steady state, resulting from 100% shock to long-term collateral requirement.

11.1.1 Financial Sector

The real sector effects of LSAPs are not discussed but are presented in Figure 5. However, the results in terms of real activity are consistent with those found in Chen et al. (2012), Harrison (2012), and Falagiarda and Saia (2013). To be clear, the goal is not to replicate the events of the financial crisis, nor the unconventional policies that tried to remedy failing economies. The approach is purely theoretical, considering several plausible scenarios that might materialise from intervention in long-term bond markets by the central bank. The initial values for the collateral requirement, before applying the shock, are $\kappa^s = 0.5$ and $\kappa_t^l = 0.5$. A shock to κ_t^l in the magnitude of approximately 0.5 was imposed, while κ^s was fixed. The shock implies both an increase in the size of the balance sheet of the central bank, in addition to a greater allotment of long-term bonds available as eligible collateral.

11.1.1.1 Deposit Bank First, deposit banks experience an increase in profitability as a result of increased deposits from households and a greater volume of loans extended. This expansionary shock increases the repayment rate for merchant banks, indicating that LSAPs have the potential to increase interbank activity. However, a decrease in interbank rates mean that, although the deposit bank is extending more credit, the price of that credit has declined. Profit made in the first few quarters is supported by the increase in deposits made by households, as the LSAP program increases their relative wealth, which affords them the opportunity to deposit money at the bank.

11.1.1.2 Firm Second, borrowing conditions are easier for firms under a large-scale asset purchase program, as the amount of acquirable reserves are increasing, with merchant banks now allowed to offer up more of their previously illiquid debt as collateral (Niestroj et al., 2013). This is reflected in the significant decrease of the credit rate. In addition to an increase in loans to firms, there is a decrease in the probability of default on these loans.

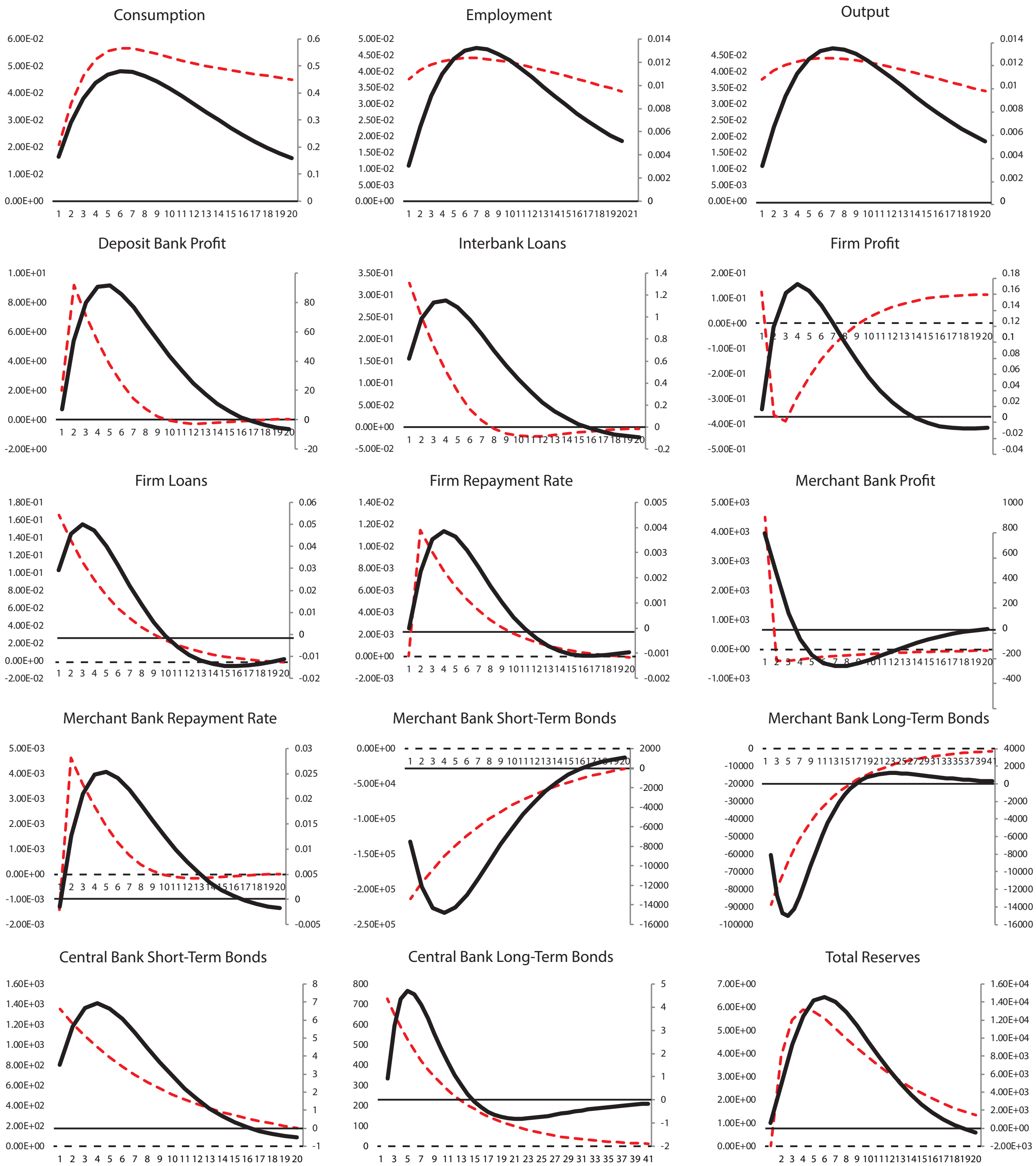
11.1.1.3 Merchant Bank Third, the merchant bank gains the most from the increase in κ^l, with profitability improving significantly in the wake of the shock. Imposing the shock significantly alters the behaviour of these banks, causing them to sell the majority of their holdings of long-term bonds - as can be seen by the decrease in long-term bonds on their balance sheets - in return for reserves. One could argue that this enacts a portfolio balance effect, with merchant banks moving their asset holdings away from long-term bonds. This effect spreads to several asset markets, moving beyond local supply effects (Dai et al., 2013). Newly issued, and total reserves, are increasing, which reflects the exchange of long-term bonds for reserves.

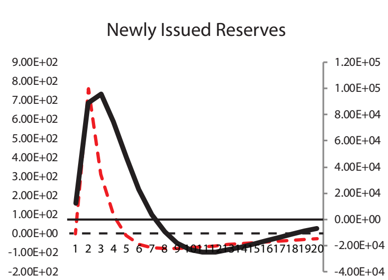
A significant portion of these long-term bonds are sold and then used to extend credit to firms. This process entails converting long-term bonds to reserves, which can be seen in the increase in total liabilities at the central bank. A portion of merchant bank reserves are then used to fuel an extension of loans. In addition, the merchant bank decides to trade a large portion of its short-term bonds in the same manner as its long-term securities. The central bank has achieved its goal, with more activity in the interbank markets as well as relaxation of the borrowing conditions for firms. In addition, the long-term rate has been depressed, which further improves borrowing conditions.

11.1.1.4 Central Bank Finally, the composition of the central bank balance sheet has certainly been affected through the application of this shock. It now holds more liabilities, primarily against long-term bonds. Gertler and Karadi (2013) refer to this as central bank intermediation, in that the central bank has "financed its asset purchases with variable interest-bearing liabilities". The benefit of central bank intermediation in this setting is that it is effectively limitless, as the central bank liabilities are essentially government debt. Having purchased these long-term securities off the balance sheet of the merchant bank, it now holds a significant portion of the long-term debt from the private sector, as also found in the work of Falagiarda and Saia (2013). In addition, the central bank has accepted some short-term debt in return for reserves.

11.1.1.5 Interest Rates The long-term interest rate has been reduced at the hand of the LSAP program, which is one of the intended goals of implementing the policy initiative (Falagiarda and Saia, 2013). All other interest rates, except the policy rate[55], experience a negative shock as a result of the portfolio balance effect generated from the expansionary policy (Dai et al., 2013). Unsurprisingly, the long-term rate is depressed the most of all the affected interest rates, as it is most closely tied to the increased purchase of long-term securities. This translates into a flattening of the yield curve and signifies a decrease in the term premium associated with the long-term rate, similar to the result of Harrison (2012) and Falagiarda and Saia (2013).

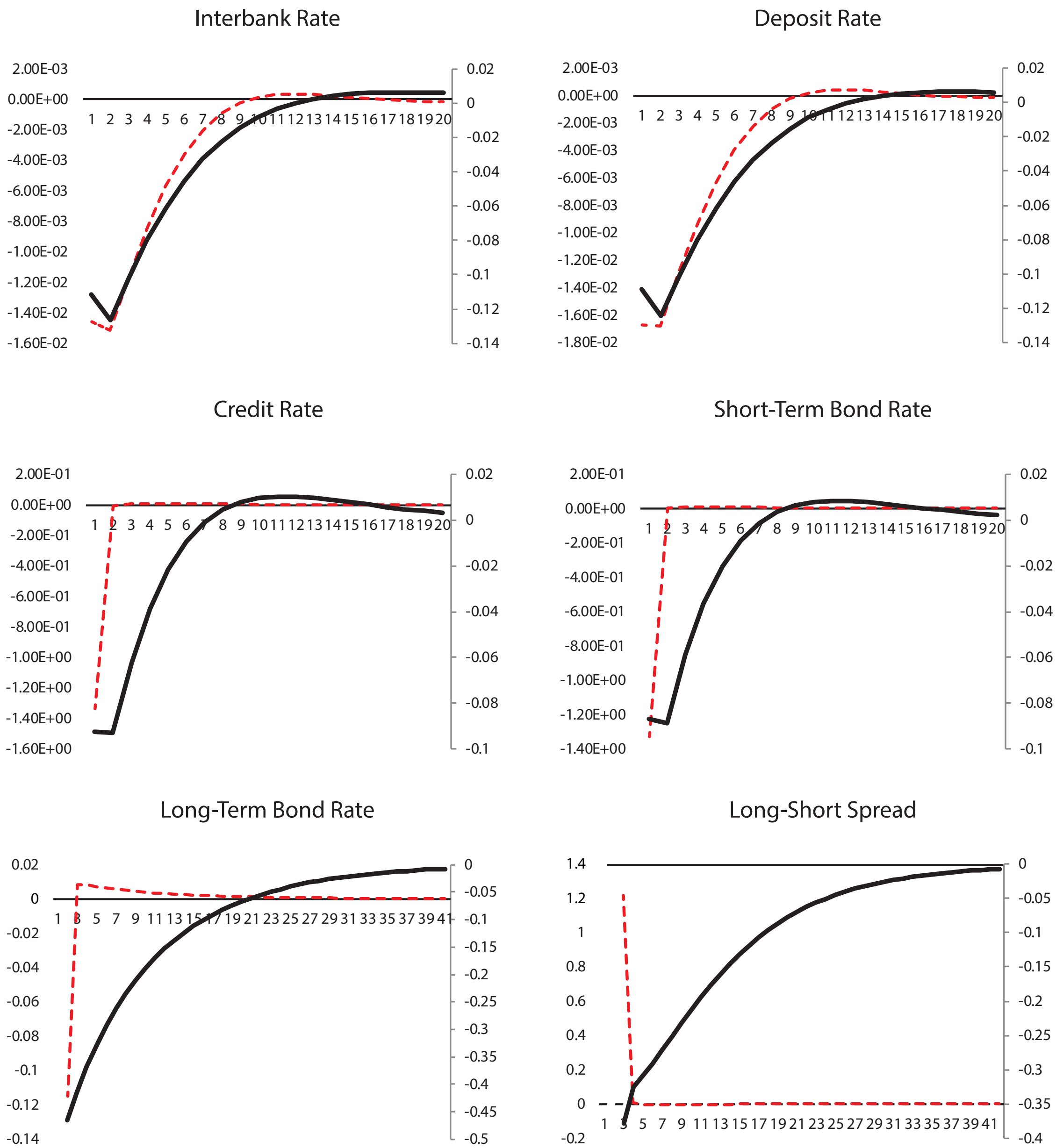
Figure 7: LSAPs vs. Increase in long-term bond growth: Real and financial sector





Note: Y-axis values represent percentage deviation from steady state.
The black line (right axis) is from a 100% shock to long-term collateral requirement, while the red dotted line (left axis) is from a shock to the long-term bond growth rate.

Figure 8: LSAPs vs. Increase in long-term bond growth: Interest rates



Note: Y-axis values represent percentage point deviation from steady state.
The black line (right axis) is from a 100% shock to long-term collateral requirement,
while the red dotted line (left axis) is from a shock to the long-term bond growth rate.

11.2 Comparisons with LSAPs

11.2.1 Large-scale Asset Purchases vs Increase in Long-term Bonds

An alternative way to think about large-scale asset purchases, is simply to increase the overall stock of long-term bonds in the economy. In order to generate a meaningful impact from the asset purchases, some agents need to have a preference for long-term bonds in order to establish imperfect asset substitutability. In this case, central banks can influence the relative supply of assets, reducing long-term security scarcity. I expect private sector agents to balance their portfolios toward the scarce asset, buying short-term bonds in order to obtain more of the scarce resource. However, this will depend on the preferences of agents in our model. To that end, the ad hoc 'preference' structure for the merchant bank will be one where short- and long-term bonds serve equally well as collateral. In reality, I did not explicitly model preferences. However, I approximated them by altering the ability of merchant banks[56] to convert bonds of differing maturities into reserves. This result is compared with the LSAP scenario of the previous section.

11.2.1.1 Shock to growth of B_t^{TL} with $\kappa^s = 0.5$ and $\kappa^l = 0.5$ In this scenario the merchant bank can transform both short- and long-term bonds into reserves with the same relative ease. In comparison with the results from the previous section, it is clear that in both cases the direction of the movement in the selected values is almost identical. Only a few differences were recorded. First, in the case of the long-term bond growth shock, the merchant banks have a sharper initial increase in the demand for liquidity. This is the result of the direct injection of reserves. Second, the overall movement of variables in the case of the haircut shock is sustained across a longer period. In particular, the long-term bond shock generates a sharp, brief, impact on interest rates, while LSAPs produces a smoother transition. This can be attributed to the autoregressive nature of the haircut process, with a high level of persistence imposed.

Third, firm profit decreases in the case of a long-term bond shock, while the opposite is true in the alternative scenario. This could be a result of more loans extended by merchant banks under the LSAP scenario. Finally, the long-short interest rate spread in the case of LSAPs is negative, while the spread is positive in the alternative. In addition, a greater spread variability between market rates is generated by the shock to the collateral requirement on long-term bonds.

While there are some minor differences, the overall results are highly similar. Importantly, this indicates that the mechanism presented in this model can generate effects similar to those already established in the literature (i.e. the long-term bond growth innovation). It carries the

[55] I believe this is the result of the increase in output, with the feedback mechanism from the Taylor Rule causing the policy rate to increase.

[56] Could be perceived as our preferred habitat investor.

benefit of allowing collateralised lending to occur naturally within the confines of the model, as opposed to a forced exogenous injection of liquidity. Merchant banks are left with more options with respect to balance sheet actions performed by the central bank.

11.2.2 Reserve vs Quasi-debt Management Policies

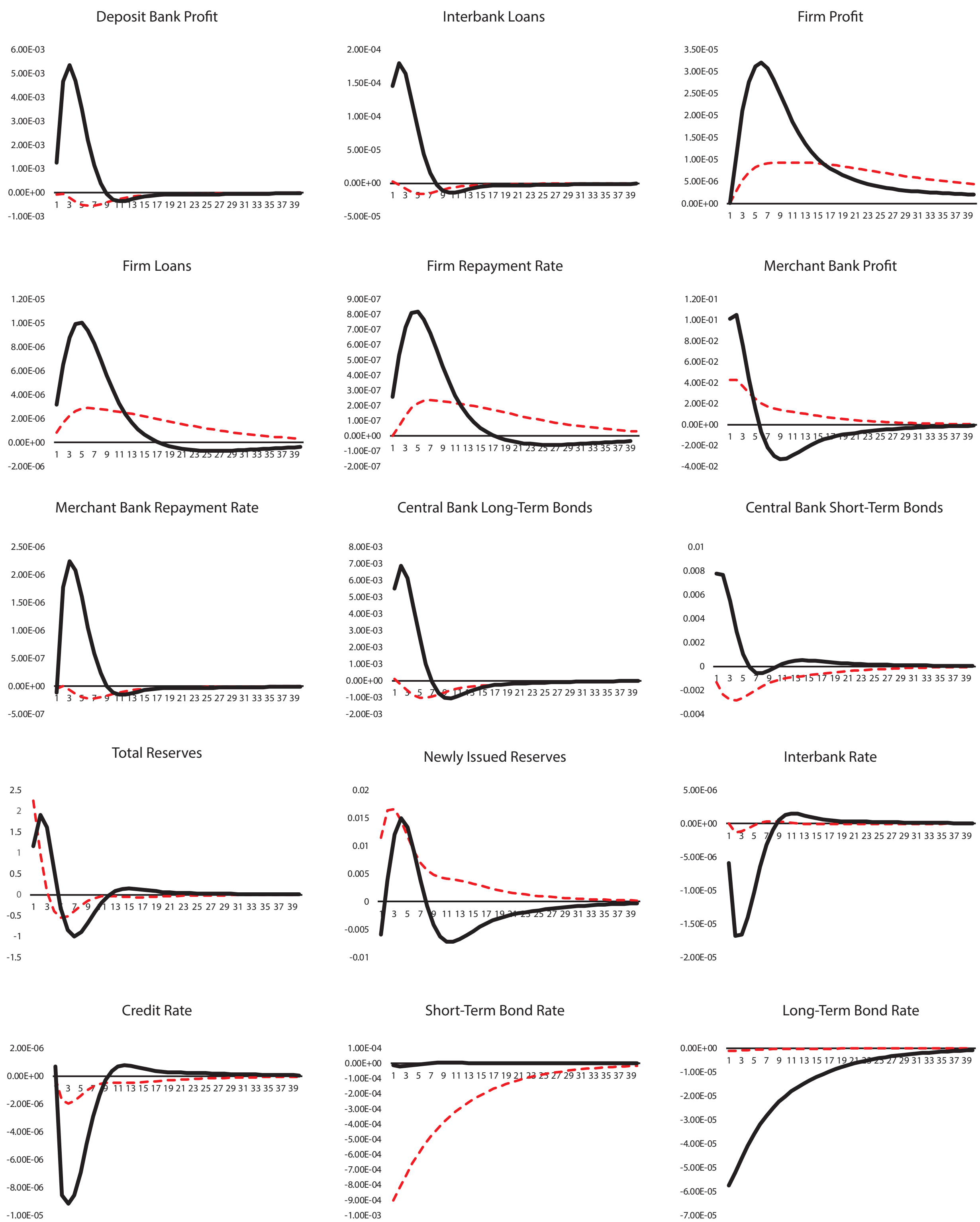
Using the complete framework affords the ability to draw a comparison between the reserve-supply ('pure' QE) model from Chapter 6 and the quasi-debt management model in this chapter. In order to conduct this comparison, the following initial values for, and shocks to κ^s and κ^l are used. First, $\kappa_t^s = 0.5$, as in Chapter 6, but κ^l is now fixed to equal 0.01. The value of κ^l is allowed to be close - but not equal - to zero, as this leads to indeterminacy in the model. In this setup there are virtually no long-term bonds held on the balance sheet of the central bank, which is similar to the model of Chapter 6. In this first scenario, a shock is imposed on κ_t^s, equal to that of Chapter 6, which generates $\kappa^s + \kappa^l \approx 1$. In essence this is a balance sheet expansion with short-term bonds offered exclusively as eligible collateral. Christensen and Krogstrup (2016) argue that this type of 'pure' QE delivers a reserve-induced portfolio balance effect.

In the comparative scenario, an initial value of $\kappa^s = 0.5$ was fixed. However, in this instance a shock was imposed on κ_t^l, which has an initial value of $\kappa^l = 0.01$. The shock imposed raises the value of κ^l close to 0.5, which means that the combined post shock value is $\kappa^s + \kappa^l \approx 1$. With this shock the central bank is implementing a change in the composition of its balance sheet, which initially contained only short-term bonds, to now incorporate long-term bonds. From this comparison one could potentially distinguish the reserve and supply-induced portfolio balance effects from each another, as defined in Christensen and Krogstrup (2016)[57]. Only the important differences observed from these shocks are discussed and depicted in Figure 9.

To reiterate, the different scenarios allow the identification of two types of balance sheet policies: one results in only short-term assets held on the balance sheet of the central bank, while the other comprises a mix of short- and long-term securities. From Figure 9, one can see that the most observable differences across these scenarios are primarily related to the relative movement of interest rates in the economy. With the introduction of long-term securities on the balance sheet of the central bank, the long-term rate reacts more sharply, exhibiting stronger downward movement. As discussed in Chapter 4, this idea is supported in the literature, as it was one of the primary goals of implementing LSAPs. In other words, the yield curve is sufficiently flattened by the introduction of long-term bonds. This reduction in the long-term rate gets transmitted to other market rates, with the interbank, credit and deposit rates all

[57]These channels are discussed in Chapter 2, under the discussion of the portfolio balance channel. I do not attempt to assign a weight to any of the channels in this thesis.

Figure 9: Reserve vs Quasi-debt management policies



Note: Y-axis values represent percentage (point) deviation from steady state.
The black solid line is from a 100% shock to long-term collateral requirement,
while the red dotted line is 100% shock to short-term collateral requirement.

significantly lower in the second scenario, with long-term bonds.

Another important result is that greater interbank and firm lending is generated in the long-term asset purchase scenario. This translates into better merchant bank and firm repayment rates, which means greater financial stability. Deposit bank and firm profit is also increased, while merchant bank profit is similar across these scenarios. Finally, while the level of reserves in the economy increases, the balance sheet of the central bank in the quasi-debt management scenario contains a greater selection of short and long-term bonds.

From these results it can be concluded that LSAPs might have the added benefit of being able to relax market conditions through their impact on interest rates. This is corroborated by the work of Cahn et al. (2014), who find "that lengthening the maturity of LTROs helps relax the bankers incentive constraint above and beyond the direct effect of short-term liquidity injections". Liquidity provided with the acquisition of short-term bonds is still invaluable, however, and its impact should not be overlooked. The question as to the relative contribution of increasing reserves versus purchasing illiquid assets is an interesting one, which could be a topic for future research, as pointed out by Kandrac and Schlusche (2015) and Christensen and Krogstrup (2016).

11.3 Contractionary Interest Rate Policy

In this section I follow on from the discussion in the previous chapter, looking now at changing the composition of the balance sheet[58]. Changes in the composition of central bank assets is represented in two scenarios, with the selection of different values for the haircut parameters under the control of the central bank. First, the baseline scenario is presented, with a 10% haircut on long-term bonds and an accompanying 90% haircut on short-term bonds. This setup is closest to the model from the previous section where $\kappa^s = 1$ and $\kappa^l = 0$, which is why I look to it as a point of departure. Another way to frame this scenario is to think in terms of the preference of the central bank. In this scenario the central bank welcomes the fact that the merchant bank considers short-term securities a convenient way to access reserves, providing the central bank with more short-term bonds in return for its unique form of liability.

Second, I developed a scenario which entails a 10% haircut on short-term bonds in relation to a sterilising 90% haircut on long-term bonds. This action changes the overall risk profile of the merchant and central banks (and, by extension, the government). In this scenario the merchant bank finds it easier to part with longer-term securities, which reduces the riskiness associated with the assets on the merchant bank's balance sheet. Conceptually, these assets are modelled

[58]Looking at a contractionary monetary policy shock gives us grounds for comparison with the previous chapter.

to resemble long-term government debt, as well as MBS or agency debt, in that they are more risky than short-term bonds. One can think of this scenario as being similar to LSAPs, with the central bank indicating a preference for riskier longer-term securities (making them easier to convert to liquidity). This is regarded as an endogenous form of LSAPs, because one expects firms to sell more of their longer-term bonds under this scenario, with the central bank being able to steer the quantity of long-term bonds through its usage of the relevant parameters in the collateralised lending mechanism.

11.3.1 Baseline: $\kappa^s = 0.9$ and $\kappa^l = 0.1$

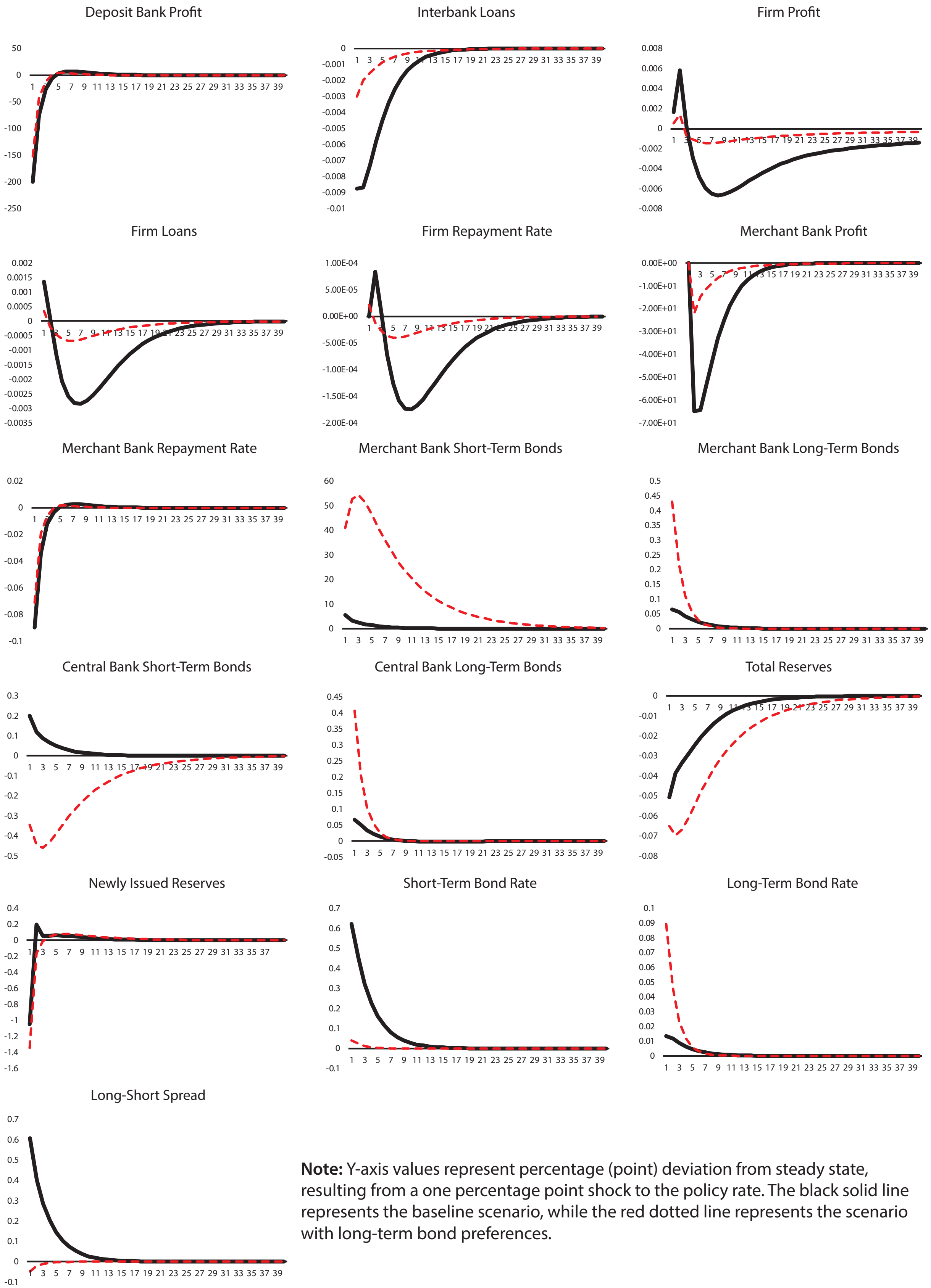
The first scenario places a greater emphasis on the central bank's issue of short-term bonds to commercial banks. I consider this to be the baseline case, as it most closely resembles the functioning of central bank intermediation as depicted in Chapter **??**. Collateralised open market operations are usually conducted on the basis of a trade between short-term government bonds and central bank liabilities (reserves). In addition to normal open market operations, the merchant bank is allowed access to central bank liabilities by offering up a fraction of its illiquid long-term bonds. Under the current constraint imposed, it is more convenient to obtain reserves by offering short-term bonds as collateral. It is also necessary to state that long-term bonds are higher yielding (higher interest rate) than reserves and short-term bonds. The reason for this being that there is a term premium associated with holding this asset, making it inherently more risky.

I identify the firm, deposit bank and merchant bank activities as being of particular importance in this analysis. Therefore, they are each discussed in turn, providing a template for discussion that I will utilise for the different scenarios. Refer to the IRFs from Figure 10 for the ensuing exposition.

First, the credit rate decreases initially in response to the increase in the policy rate. This result is counter-intuitive, as we would expect a positive pass-through. However, it is short-lived and represents the only sign reversal among the different market rates[59]. After the first period decline, the credit rate becomes positive, which is more in line with expectations. As a result of the behaviour of the credit rate, firm profits initially increase, but then fall below the pre-shock value in response to the monetary policy shock. The eventual decline in profitability is similar to the result from the setup without long-term bonds, where $\kappa^l = 0$ and $\kappa^s = 1$. Loans extended to the firm increase in the first quarter after the shock, but then decrease by a substantial amount, which is in line with the behaviour of the credit rate. The repayment rate on loans exhibits an

[59]The fact that the deviation is so brief leads me to believe that it is an artefact in the model, rather than an explicit feature.

Figure 10: Contractionary interest rate policy



Note: Y-axis values represent percentage (point) deviation from steady state, resulting from a one percentage point shock to the policy rate. The black solid line represents the baseline scenario, while the red dotted line represents the scenario with long-term bond preferences.

initial increase, but then we have a steep decrease in repayment as the firms respond to the rise in the credit rate.

Second, the policy shock has a significant negative impact on interbank loans. In addition to the policy rate impact, the decrease in interbank loans can be explained by the increase in demand for short- and long-term bonds on the part of the merchant bank, indicating a substitution away from interbank loans to relatively abundant and easily convertible government securities. The pass-through of the policy rate is greater to the short- than the long-term rate. This is in line with expectations, considering the structure of the collateralised borrowing mechanism, with the haircut allowing a greater transfer of the shock to the short-term rate. Repayments on interbank loans declines following the shock, as a result of the increased cost of borrowing. Deposit bank profit suffers at the hand of the policy shock and takes several quarters to return to normal. Furthermore, merchant bank profit decreases sharply following the shock, and remains low for a few quarters.

Finally, central bank liabilities decrease initially given the increase in the policy rate. Merchant banks buy relatively cheap short- and long-term debt, which increases the quantity of both assets on their portfolio, while decreasing the amount reserves held. This highlights the action of commercial banks given an increased availability of investment vehicles. Ultimately, the central bank holds a greater absolute amount of short- and long-term bonds on its balance sheet in this scenario.

Interest rates on assets eligible for collateral carry the greatest weight of the pass-through from the shock. The short-term bond rate does not fully absorb the movement in the policy rate[60], but is more reactive than the long-term bond rate. There is a liquidity premium placed on short-term bonds in this case. Both of these rates are above the deposit, credit and interbank loan rates, which reflects the spread on bonds offered as collateral versus those that are not.

11.3.2 LSAPs: $\kappa^s = 0.1$ and $\kappa^l = 0.9$

This scenario is similar to the preferred habitat approach, in that investors 'prefer' to invest in longer-term assets. In this scenario merchant banks face a limited supply of money in return for their short-term bonds, relative to long-term bonds. This means that short-term bonds are limited in their capacity to be converted to central bank liabilities. Converting long-term bonds to money (an illiquid to liquid asset) is facilitated by a lower haircut on long-term bonds. This is quite similar to the LSAP programs implemented by the US in the first few rounds of so-called quantitative easing. I compare the results to those of the first (baseline) scenario.

[60] In this scenario the magnitude of the increase in the short-term bond rate is just above 60% of the movement in the policy rate.

First, firms behave similarly in the different scenarios posed, with the most obvious difference being in the repayment rate of the firm. Under increased long-term bond purchases, the repayment rate decreases more gradually and remains persistently below the pre-shock value. In addition, the credit rate also moves less severely in this instance, with the initial drop being more moderate. Second, interbank loans once again decrease, but by a smaller magnitude. This means that merchant banks are increasing their exposure to the interbank market in this scenario. One interpretation would be that these merchant banks are finding it more lucrative to access wholesale funding than to exchange their long-term bonds for reserves. It might also be that long-term bonds are yielding such a high return that it is not worth trading them in. In addition, the deposit bank makes slightly smaller losses in this scenario.

Third, the lower interbank rate translates into an associated decrease in merchant bank default. Although the impact from the change is small in terms of the merchant bank default, it is important. It indicates that inducing sales of long-term bonds to the central bank has resulted in a slight decrease in the default rate. This does not mean that merchant banks got rid of all their long-term securities, but a large portion ended up on the balance sheet of the central bank. In addition, the total level of short-term bonds held by the central bank decreases, with merchant banks looking to purchase short-term bonds, but being constrained by the haircut imposed.

Ultimately, in this setup the central bank balance sheet contains more long-term securities and fewer short-term bonds. The overall level of liabilities decreases as merchant banks appear to favour securities over reserves. The increased long-run interest rate sensitivity dictates that the increase in the policy rate will have a greater effect on the long-term rate than in the previous scenario.

12 Chapter Conclusion

In developing the model for this chapter, I attempted to address some of the shortcomings of modern DSGE models. Several attractive features are combined into one model, with the purpose of looking at changes in the size of the balance sheet of the central bank. Some of the important amendments to the traditional New-Keynesian DSGE model are endogenous default, an interbank market with heterogeneous banks, interest rate spreads and a role for balance sheet policies. In particular, the addition of the collateralised borrowing mechanism from Schabert (2015) provides an elegant way to present endogenous changes in the size of the balance sheet of the central bank.

Central banks, in the model, have the ability to inject liquidity into the financial system through open market operations. Injections are specifically directed at the merchant bank, as dictated by the collateralised borrowing constraint. Merchant banks, are similar, in reality, to commercial banks with direct access to central bank reserves. As depicted by an increase in the value of κ_t, a balance sheet expansion results in improved borrowing conditions for these banks, which, in turn, improves interbank market activity and reduces firm and interbank loan default. The finding from this chapter illustrates that expanding the balance sheet of the central bank can have system-wide implications, especially with respect to the agents responsible for credit extension to the broader economy.

In addition, central banks have full control over the haircut mechanism, κ, which affords them the ability to alter the size of the balance sheet autonomously. It is plausible that in some scenarios this will allow them to implement interest rate policy in conjunction with balance sheet policy, amplifying, or perhaps detracting from, the effect of interest rate policy. As illustrated in the last part of the chapter, the central bank can increase the interest rate and decide on varying degrees of balance sheet size, dependent on the value of κ. Smaller values of κ, for example, will impose stricter lending conditions and reduce the availability of credit in the system. In the next chapter, I determine what effect changes in the composition of the balance sheet might have on financial stability and economic activity.

13 Chapter Conclusion

In this chapter I have included long-term bonds available for collateral in open market operations. The introduction of these bonds in a non-trivial way affords the central bank the opportunity to change the composition of the assets held on its balance sheet, with a significant impact on the financial markets and the real sector. In particular, the introduction of long-term government bonds draws a comparison to quasi-debt management operations, such as those conducted by the Fed by means of the Maturity Extension Program (MEP). It is clear from the scenarios presented here that adding long-term bond purchases further assists, beyond that of reserve-supply policy, in providing liquidity and securing financial stability. The primary mechanism through which the introduction of long-term bonds is realised is in lowering the long-term rate, which results in easier borrowing conditions for the broader economy.

In comparison with the previous chapter, which entailed only the purchase of short-term assets, the economic conditions were relaxed even further under long-term bond purchases, with the effect most clearly represented in a reduction of the price of the asset targeted. With large-scale asset purchases being potentially more beneficial to financial stability than ‘pure’ QE, they might be used in a complementary fashion with interest rate and macroprudential policy to combat the build-up of financial imbalances in specific sectors, such as the mortgage market. This does, however, increase the financial market footprint of the central bank, which should be considered before conducting such a policy. Financial market exposure notwithstanding, targeted purchases could present a sharper instrument than the short-term policy rate.

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A Log-Linearised Model

A.1 Example

In order to illustrate the approach followed to linearise the model in this thesis, an example is provided with respect to marginal utility of consumption. This is the FOC for the household w.r.t consumption,

$$(C_t - hC_{t-1})^{-\sigma_c} - h\beta\mathbb{E}_t(C_{t+1} - hC_t)^{-\sigma_c} = \lambda_t^h$$

The first step is to define $\Omega_t = (C_t - hC_{t-1})$, which gives us,

$$(\Omega_t)^{-\sigma_c} - h\beta\mathbb{E}_t(\Omega_{t+1})^{-\sigma_c} = \lambda_t^h$$

Using Uhlig's method, log-linearisation gives,

$$(\Omega_{ss})^{-\sigma_c}(1+\hat{\Omega}_t)^{-\sigma_c} - (\Omega_{ss})^{-\sigma_c}h\beta\mathbb{E}_t(1+\hat{\Omega}_{t+1})^{-\sigma_c} = \lambda^h_{ss}(1+\hat{\lambda}^h_t)$$
$$(\Omega_{ss})^{-\sigma_c}(1-\sigma_c\cdot\hat{\Omega}_t) - (\Omega_{ss})^{-\sigma_c}h\beta\mathbb{E}_t(1-\sigma_c\cdot\hat{\Omega}_{t+1})^{-\sigma_c} = \lambda^h_{ss}(1+\hat{\lambda}^h_t)$$

Using the fact that in steady state $(\Omega_{ss})^{-\sigma_c}(1-h\beta) = \lambda^h_{ss}$, we have that,

$$\sigma_c\Omega_{ss}^{-\sigma_c}h\beta\mathbb{E}_t(\hat{\Omega}_{t+1}) - \sigma_c\Omega_{ss}^{-\sigma_c}\hat{\Omega}_t = \lambda^h\hat{\lambda}^h_t$$
$$\sigma_c\left[h\beta\mathbb{E}_t(\hat{\Omega}_{t+1}) - \hat{\Omega}_t\right] = (1-h\beta)\hat{\lambda}^h_t$$

Next we can log-linearise, $\Omega_t = (C_t - hC_{t-1})$.

$$\Omega_{ss}(1+\hat{\Omega}_t) = C_{ss}(1+\hat{C}_t) - h\cdot C_{ss}(1+\hat{C}_{t-1})$$

Using the steady state $\Omega_{ss} = C_{ss} - hC_{ss}$ we can reduce the equation to,

$$\Omega_{ss}\hat{\Omega}_t = C_{ss}\hat{C}_t - h\cdot C_{ss}\hat{C}_{t-1}$$
$$\therefore \quad \hat{\Omega}_t = \frac{C_{ss}}{\Omega_{ss}}\left[\hat{C}_{t-1} - h\cdot\hat{C}_t\right]$$

Substitute this variable to get,

$$\sigma_c\left[h\beta\mathbb{E}_t\left(\frac{C_{ss}}{\Omega_{ss}}\left[\hat{C}_{t+1} - h\cdot\hat{C}_t\right]\right) - \left(\frac{C_{ss}}{\Omega_{ss}}\left[\hat{C}_t - h\cdot\hat{C}_{t-1}\right]\right)\right] = (1-h\beta)\hat{\lambda}^h_t$$
$$\sigma_c\left[h\beta\mathbb{E}_t\left(\frac{C_{ss}}{C_{ss}-hC_{ss}}\left[\hat{C}_{t+1} - h\cdot\hat{C}_t\right]\right) - \left(\frac{C_{ss}}{C_{ss}-hC_{ss}}\left[\hat{C}_t - h\cdot\hat{C}_{t-1}\right]\right)\right] = (1-h\beta)\hat{\lambda}^h_t$$

We can use some algebra to simplify this equation,

$$\sigma_c\left[h\beta\mathbb{E}_t\left(\frac{1}{1-h}\left[\hat{C}_{t+1} - h\cdot\hat{C}_t\right]\right) - \left(\frac{1}{1-h}\left[\hat{C}_t - h\cdot\hat{C}_{t-1}\right]\right)\right] = (1-h\beta)\hat{\lambda}^h_t$$
$$\left(\frac{h\beta\sigma_c}{1-h}\right)\mathbb{E}_t(\hat{C}_{t+1}) - \left[\frac{(1+h^2\beta)\sigma_c}{1-h}\right]\hat{C}_t + \left(\frac{h\sigma_c}{1-h}\right)\hat{C}_{t-1} = (1-h\beta)\hat{\lambda}^h_t$$

The work below shows the complete linearised version of the model for Chapter 6.

A.2 Household

A.2.1 Euler Equation

The Euler equation is,

$$h\beta^h \mathbb{E}_t \left[\frac{(C_{t+1} - hC_t)^{-\sigma_c}}{\pi_{t+1}} \right] = \frac{(C_t - hC_{t-1})^{-\sigma_c}}{R_t^d}$$

Log-linearisation delivers,

$$\begin{aligned} &\frac{h}{R_{ss}^d (C_{ss} - hC_{ss})^{\sigma_c}} \left[\frac{\sigma_c}{(1-h)} (\hat{C}_t - \hat{C}_{t-1}) + \hat{R}_t^d \right] \\ &= \frac{h\beta^h}{\pi_{ss} (C_{ss} - hC_{ss})^{\sigma_c}} \left[\frac{\sigma_c}{(1-h)} (\hat{C}_{t+1} - \hat{C}_t) + \hat{\pi}_{t+1} \right] \end{aligned} \tag{77}$$

A.2.2 Wage Setting

The next equation to consider is the law of motion of f_t. The first part of this equation is given by,

$$f_t = \frac{\eta - 1}{\eta} (w_t^*)^{1-\eta} \lambda_t^h (w_t)^\eta N_t + \beta^h \theta_w \mathbb{E}_t \left(\frac{(\pi_t)^{\tau_w}}{\pi_{t+1}} \right)^{1-\eta} \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\eta - 1} f_{t+1}$$

Linearising the equation starts as follows,

$$\begin{aligned} f_{ss} \exp^{\hat{f}_t} = &\frac{\eta - 1}{\eta} (w_{ss}^*)^{1-\eta} \lambda_{ss}^h (w_{ss})^\eta N_{ss} \, \exp^{(1-\eta)\hat{w}_t^* + \hat{\lambda}_t^h + \eta \hat{w}_t + \hat{N}_t} \\ &+ \beta \theta_w \pi_{ss}^{(\tau_w - 1)(\eta - 1)} f_{ss} \mathbb{E}_t \exp^{\hat{f}_{t+1} - (1-\eta)[\hat{\pi}_{t+1} - \tau_w \hat{\pi}_t + \hat{w}_{t+1}^* - \hat{w}_t^*]} \end{aligned}$$

With simplification we get,

$$\begin{aligned} f_{ss} \hat{f}_t = &\frac{\eta - 1}{\eta} (w_{ss}^*)^{1-\eta} \lambda_{ss}^h (w_{ss})^\eta N_{ss} \left((1-\eta)\hat{w}_t^* + \hat{\lambda}_t^h + \eta \hat{w}_t + \hat{N}_t \right) \\ &+ \beta \theta_w \pi_{ss}^{(\tau_w - 1)(\eta - 1)} f_{ss} \mathbb{E}_t \left[\hat{f}_{t+1} - (1-\eta)[\hat{\pi}_{t+1} - \tau_w \hat{\pi}_t + \hat{w}_{t+1}^* - \hat{w}_t^*] \right] \end{aligned}$$

In steady state we have that, $f_{ss} - \beta \theta_w f_{ss} \pi_{ss}^{(\tau_w - 1)(1-\eta)} = \frac{\eta - 1}{\eta} (w_{ss})^{1-\eta} \lambda_{ss}^h w_{ss}^\eta N_{ss}$, which we can use to simplify the equation above. The final linearised equation is then,

$$\begin{aligned} \hat{f}_t = &\left(1 - \beta \theta_w \pi_{ss}^{(\tau_w - 1)(1-\eta)}\right) \left((1-\eta)\hat{w}_t^* + \hat{\lambda}_t^h + \eta \hat{w}_t + \hat{N}_t \right) \\ &+ \beta \theta_w \pi_{ss}^{(\tau_w - 1)(\eta - 1)} \mathbb{E}_t \left[\hat{f}_{t+1} - (1-\eta)[\hat{\pi}_{t+1} - \tau_w \hat{\pi}_t + \hat{w}_{t+1}^* - \hat{w}_t^*] \right] \end{aligned} \tag{78}$$

The second part of the law of motion for f_t is,

$$f_t = \left(\frac{w_t}{w_t^*}\right)^{\eta(1+\sigma_n)} (N_t)^{(1+\sigma_n)} + \beta\theta_w \mathbb{E}_t \left(\frac{(\pi_t)^{\tau_w}}{\pi_{t+1}}\right)^{-\eta(1+\sigma_n)} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\eta(1+\sigma_n)} f_{t+1}$$

We start by linearising the equation in the following way,

$$f_{ss}\exp^{\hat{f}_t} = \left(\frac{w_{ss}}{w_{ss}^*}\right)^{\eta(1-\sigma_m)} (N_{ss})^{1+\sigma_n} \exp^{\eta(1+\sigma_n)[\hat{w}_t - \hat{w}_t^*]+(1+\sigma_n)\hat{N}_t}$$
$$+\beta\theta_w(\pi_{ss})^{\eta(1+\sigma_n)(1-\tau_w)} f_{ss}\mathbb{E}_t \exp^{\hat{f}_{t+1}+\eta(1+\sigma_n)[\hat{\pi}_{t+1}-\tau_w\hat{\pi}_t+\hat{w}_{t+1}^*-\hat{w}_t^*]}$$

From this it can be shown that,

$$f_{ss}\hat{f}_t = \left(\frac{w_{ss}}{w_{ss}^*}\right)^{\eta(1-\sigma_m)} (N_{ss})^{1+\sigma_n} \left(\eta(1+\sigma_n)[\hat{w}_t - \hat{w}_t^*] + (1+\sigma_n)\hat{N}_t\right)$$
$$+\beta\theta_w(\pi_{ss})^{\eta(1+\sigma_n)(1-\tau_w)} f_{ss}\mathbb{E}_t \left(\hat{f}_{t+1} + \eta(1+\sigma_n)[\hat{\pi}_{t+1} - \tau_w\hat{\pi}_t + \hat{w}_{t+1}^* - \hat{w}_t^*]\right)$$

In steady state we have that, $f_{ss} - \beta\theta_w f_{ss}(\pi_{ss})^{\eta(1+\sigma_n)(1-\tau_w)} = \left(\frac{w_{ss}}{w_{ss}^*}\right)^{\eta(1-\sigma_m)} (N_{ss})^{1+\sigma_n}$, which we can use to simplify the equation above. The final linearised equation is then,

$$\hat{f}_t = \left(1 - \beta\theta_w(\pi_{ss})^{\eta(1+\sigma_n)(1-\tau_w)}\right) \left(\eta(1+\sigma_n)[\hat{w}_t - \hat{w}_t^*] + (1+\sigma_n)\hat{N}_t\right)$$
$$+\beta\theta_w(\pi_{ss})^{\eta(1+\sigma_n)(1-\tau_w)}\mathbb{E}_t \left(\hat{f}_{t+1} + \eta(1+\sigma_n)[\hat{\pi}_{t+1} - \tau_w\hat{\pi}_t + \hat{w}_{t+1}^* - \hat{w}_t^*]\right) \tag{79}$$

A.2.3 Real Wage Index

In addition to the law of motion for f_t we have the real wage index, given by,

$$1 = \theta_w \left(\frac{(\pi_{t-1})^{\tau w}}{\pi_t}\right)^{1-\eta} \left(\frac{w_{t-1}}{w_t}\right)^{1-\eta} + (1-\theta_w)(\pi_t^{w^*})^{1-\eta}$$

This equation can be written as,

$$1 = \theta_w \pi_{ss}^{(\tau_w - 1)(1-\eta)} \exp^{-(1-\eta)(\hat{\pi}_t - \tau_w\hat{\pi}_{t-1} + \hat{\pi}_t^w)} + (1-\theta_w)(\pi_{ss}^{w^*})^{1-\eta} \exp^{(1-\eta)\hat{\pi}_t^{w^*}}$$

Which can then be log-linearised, to give,

$$\theta_w \pi_{ss}^{(\tau_w-1)(1-\eta)} (\hat{\pi}_t - \tau_w\hat{\pi}_{t-1} + \hat{\pi}_t^w) = (1-\theta_w)(\pi_{ss}^{w^*})^{1-\eta}\, \hat{\pi}_t^{w^*}$$

This provides us the final linearised equation,

$$\frac{\theta_w \pi_{ss}^{(\tau_w - 1)(1-\eta)}}{(1-\theta_w)(\pi_{ss}^{w^*})^{1-\eta}} \left(\hat{\pi}_t - \tau_w \hat{\pi}_{t-1} + \hat{\pi}_t^w\right) = \hat{w}_t^* - \hat{w}_t \tag{80}$$

A.3 Firm

A.3.1 Labour

The first order condition with respect to labour is,

$$w_t = (1-\alpha) K_t^{\alpha} N_t{}^{-\alpha}$$

This can be log-linearised to give,

$$\hat{w}_t = \alpha \hat{K}_t - \alpha \hat{N}_t \tag{81}$$

A.3.2 Capital

The first order condition with respect to capital is,

$$\lambda_t^f - \beta^f \mathbb{E}_t[(1-\varphi)\lambda_{t+1}^f] = \alpha K_t^{\alpha-1} N_t^{1-\alpha}$$

This can be log-linearised to give,

$$\lambda_{ss}^f \hat{\lambda}_t^f - \beta^f \lambda_{ss}^f \mathbb{E}_t[(1-\varphi)\hat{\lambda}_{t+1}^f] = \alpha K_{ss}^{\alpha-1} N_{ss}^{1-\alpha} \left[(\alpha-1)\hat{K}_t + (1-\alpha)\hat{N}_t\right] \tag{82}$$

A.3.3 Marginal Cost

The real marginal cost function is,

$$mc_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{\lambda_t^f - \beta^f \mathbb{E}_t[(1-\varphi)\lambda_{t+1}^f]}{\alpha}\right)^{\alpha}$$

where we can redefine $r_t = \lambda_t^f - \beta^f \mathbb{E}_t[(1-\varphi)\lambda_{t+1}^f]$, which gives us,

$$mc_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^{\alpha}$$

Log-linearising this equation gives,

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}_t$$

The log-linearisation of r_t leaves us with,

$$\hat{r}_t = \frac{\hat{\lambda}^f_t - \beta^f \mathbb{E}_t[(1-\varphi)\hat{\lambda}^f_{t+1}]}{1-\beta^f(1-\varphi)}$$

This gives us the final linearisation of the marginal cost function as,

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \frac{1}{1-\beta^f(1-\varphi)}\left[\hat{\lambda}^f_t - \beta^f \mathbb{E}_t[(1-\varphi)\hat{\lambda}^f_{t+1}]\right] \tag{83}$$

A.3.4 Loans from merchant bank

The first order condition with respect to loans from the merchant bank is,

$$\begin{aligned}
&\frac{\lambda^f_t}{R^c_t}\left(1-\Gamma\left(\frac{L^b_t}{L^b_{t-1}}\right) - \Gamma'\left(\frac{L^b_t}{L^b_{t-1}}\right)\frac{L^b_t}{L^b_{t-1}}\right) \\
&= \beta^f \mathbb{E}_t\left[\frac{\psi_{t+1}}{\pi_{t+1}} - \frac{\lambda^f_{t+1}}{R^c_{t+1}}\left(\Gamma'\left[\frac{L^b_{t+1}}{L^b_t}\right]\left(\frac{L^b_{t+1}}{L^b_t}\right)^2\right)\right] + (\beta^f)^2\mathbb{E}_t\left[\omega_\psi(1-\psi_{t+1})^2 L^b_t\right]
\end{aligned}$$

Log-linearisation leads to the following equation,

$$\begin{aligned}
&\frac{\lambda^f_{ss}}{R^c_{ss}}\left(\hat{\lambda}^f_t - \hat{R}^c_t\right) - \theta\frac{\lambda^f_{ss}}{R^c_{ss}}\left(\hat{L}^b_t - \hat{L}^b_{t-1}\right) + \theta\beta^f\frac{\lambda^f_{ss}}{R^c_{ss}}\mathbb{E}_t\left(\hat{L}^b_{t+1} - \hat{L}^b_t\right) \\
&= \beta^f\frac{\psi_{ss}}{\pi_{ss}}\mathbb{E}_t\left[(\hat{\psi}_{t+1} - \hat{\pi}_{t+1})\right] + (\beta^f)^2\frac{\omega_\psi}{2}(1-\psi_{ss})^2 L^b_{ss}\left[-2\left(\frac{\psi_{ss}}{1-\psi_{ss}}\right)\hat{\psi}_{t+1} + \hat{L}^b_t\right]
\end{aligned} \tag{84}$$

A.3.5 Default

The first order condition with respect to default is,

$$\frac{L^b_{t-1}}{\pi_t} = d_\psi + \beta^f\omega_\psi\left[(1-\psi_t)(L^b_{t-1})^2\right]$$

This equation can be written in log-linear form as,

$$\frac{L^b_{ss}}{\pi_{ss}}(1+\hat{L}^b_{t-1} - \hat{\pi}_t) = d_\psi + \beta^f\frac{\omega_\psi}{2}(1-\psi_{ss})(L^b_{ss})^2\left[1-\left(\frac{\psi_{ss}}{1-\psi_{ss}}\right)\hat{\psi}_t + 2\hat{L}^b_{t-1}\right]$$

A.3.6 Price Setting

Price setting is done in the vein of Rotemberg, given by the following equation,

$$\left(\frac{\pi_t}{(\bar{\pi})^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}} - 1\right)\frac{\pi_t}{(\bar{\pi})^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}}$$
$$= \beta^f \mathbb{E}_t\left[\left(\frac{\pi_{t+1}}{(\bar{\pi})^{1-\gamma_p}(\pi_t)^{\gamma_p}} - 1\right)\frac{\pi_{t+1}}{(\bar{\pi})^{1-\gamma_p}(\pi_t)^{\gamma_p}}\frac{y_{t+1}}{y_t}\right] + \left[\frac{1-\epsilon(1+mc_t)}{\varrho}\right]$$

Log-linearisation delivers,

$$\left(\frac{(1+\hat{\pi}_t)}{(1+\gamma_p(\hat{\pi}_{t-1}))} - 1\right)\frac{(1+\hat{\pi}_t)}{(1+\gamma_p(\hat{\pi}_{t-1}))}$$
$$= \beta^f \mathbb{E}_t\left[\left(\frac{(1+\hat{\pi}_{t+1})}{(1+\gamma_p(\hat{\pi}_t))} - 1\right)\frac{(1+\hat{\pi}_{t+1})}{(1+\gamma_p(\hat{\pi}_t))}\frac{1+\hat{y}_{t+1}}{1+\hat{y}_t}\right] + \frac{1-\epsilon}{\varrho} - \frac{\epsilon[mc_{ss}(1+\hat{mc}_t)]}{\varrho}$$

Further simplification yields,

$$[\hat{\pi}_t - \gamma_p(\hat{\pi}_{t-1})] = \beta^f \mathbb{E}_t\left[\hat{\pi}_{t+1} - \gamma_p(\hat{\pi}_t)\right] + \left(\frac{1-\epsilon}{\varrho}\right)\hat{mc}_t \tag{85}$$

A.4 Deposit Bank

A.4.1 Deposits

The first FOC for the deposit bank with respect to deposits is,

$$\frac{1}{R_t^d}(\pi_t^l + 1)^{-\sigma_l} = \beta^l \mathbb{E}_t\left[\frac{1}{\pi_{t+1}}(\pi_{t+1}^l + 1)^{-\sigma_l}\right] - \Xi_t$$

Log-linearisation delivers,

$$\frac{1}{R_{ss}^d}\left[\hat{R}_t^d + \left(\frac{\pi_{ss}^l}{1+\pi_{ss}^l}\right)\sigma_l\hat{\pi}_t^l\right]$$
$$= \beta^l\frac{1}{\pi_{ss}}\left[\left(\frac{\pi_{ss}^l}{1+\pi_{ss}^l}\right)\sigma_l\hat{\pi}_{t+1}^l + \hat{\pi}_{t+1}\right] - \Xi_{ss}\hat{\Xi}_t \tag{86}$$

A.4.2 Interbank Loans

The first FOC for the deposit bank with respect to interbank loans is,

$$\frac{1}{R_t^l}(\pi_t^l + 1)^{-\sigma_l} = \beta^l \mathbb{E}_t \left[\frac{\delta_{t+1}}{\pi_{t+1}} (\pi_{t+1}^l + 1)^{-\sigma_l} \right] + \Xi_t$$

Log-linearisation of this equation delivers,

$$\begin{aligned} &\left(\frac{1}{R_{ss}^l} \right) \left[\hat{R}_t^l + \left(\frac{\pi_{ss}^l}{1 + \pi_{ss}^l} \right) \sigma_l \hat{\pi}_t^l \right] \\ &= \beta^l \left(\frac{\delta_{ss}}{\pi_{ss}} \right) \left[\hat{\pi}_{t+1} - \hat{\delta}_{t+1} + \left(\frac{\pi_{ss}^l}{1 + \pi_{ss}^l} \right) \sigma_l \hat{\pi}_{t+1}^l \right] + \Xi_{ss} \hat{\Xi}_t \end{aligned} \tag{87}$$

A.5 Merchant Bank

A.5.1 Money Holdings

The first order condition with respect to money holdings,

$$\dot{U}_t^b = \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b}{\pi_{t+1}} \right) - \Upsilon_t$$

The final log-linearised equation is,

$$\frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_t^b = \beta^b \left[\frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_{t+1}^b + \hat{\pi}_{t+1} \right] - \Upsilon_{ss} \hat{\Upsilon}_t$$

A.5.2 Newly Issued Reserves

The first order condition with respect to newly issued reserves is,

$$\dot{U}_t^b = R_t^m (\dot{U}_t^b + \eta_t)$$

This gives the following log-linearised equation,

$$\begin{aligned} &R_{ss}^m \eta_{ss} \left(\hat{R}_t^m + \hat{\eta}_t \right) \\ &= -\left((\pi_{ss}^b + 1)^{-\sigma_b} R_{ss}^m \right) \left[\hat{R}_t^m + \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_t^b \right] + (\pi_{ss}^b + 1)^{-\sigma_b} \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_t^b - \Upsilon_{ss} \hat{\Upsilon}_t \end{aligned} \tag{88}$$

A.5.3 Interbank Loans

The first order condition with respect to interbank loans,

$$\dot{U}_t^b \frac{1}{R_t^l} = \beta^b \mathbb{E}_t \left[\left(\frac{\delta_{t+1}}{\pi_{t+1}} \right) \dot{U}_{t+1}^b \right] + (\beta^b)^2 \mathbb{E}_{t+1} \left[\omega_\delta (1 - \delta_{t+1})^2 L_t^l \dot{U}_{t+2}^b \right] + \Upsilon_t$$

The final log-linearised equation is,

$$\begin{aligned}
&- \left(\frac{1}{R_{ss}^l} \right) \left[\hat{R}_t^l + \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_t^b \right] \\
&= \frac{\beta^b \delta_{ss}}{\pi_{ss}} \left[\hat{\delta_{t+1}} - \hat{\pi}_{t+1} - \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_{t+1}^b \right] \\
&+ (\beta^b)^2 \left(\frac{\omega_b}{2} \right) L_{ss}^l (1 - \delta_{ss})^2 \left[-2 \left(\frac{\delta_{ss}}{1 - \delta_{ss}} \right) \hat{\delta}_{t+1} + \hat{L}_t^l - \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_{t+2}^b \right] + \Upsilon_{ss} \hat{\Upsilon}_t
\end{aligned} \tag{89}$$

A.5.4 Loans to Firms

The first order condition with respect to loans to firms,

$$\dot{U}_t^b \frac{1}{R_t^c} = \beta^b \mathbb{E}_t \left[\frac{\psi_{t+1}}{\pi_{t+1}} \dot{U}_{t+1}^b \right] - \Upsilon_t$$

The final log-linearised equation is,

$$- \left(\frac{1}{R_{ss}^c} \right) \left[\hat{R}_t^c + \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_t^b \right] = \beta^b \left(\frac{\psi_{ss}}{\pi_{ss}} \right) \left[\hat{\psi}_{t+1} - \hat{\pi}_{t+1} - \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_{t+1}^b \right] - \Upsilon_{ss} \hat{\Upsilon}_t \tag{90}$$

A.5.5 Short Term Bonds

The first order condition with respect to bonds is,

$$\dot{U}_t^b \frac{1}{R_t^b} = \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b + \kappa \cdot \eta_{t+1}}{\pi_{t+1}} \right) - \Upsilon_t$$

The final log-linearisation is,

$$\begin{aligned}
&- \frac{1}{R_{ss}^b} (\pi_{ss}^b + 1)^{-\sigma_b} \left[\hat{R}_t^b + \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_t^b \right] \\
&= \beta^b (\pi_{ss}^b + 1)^{-\sigma_b} \frac{1}{\pi_{ss}} \left[-\hat{\pi}_{t+1} - \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_{t+1}^b \right] + \beta^b \frac{\kappa \eta_{ss}}{\pi_{ss}} \left[-\hat{\pi}_{t+1} + \hat{\kappa}_{t+1} + \hat{\eta}_{t+1} \right] - \Upsilon_{ss} \hat{\Upsilon}_t
\end{aligned} \tag{91}$$

A.5.6 Default

The first order condition with respect to default is

$$\dot{U}_t^b \frac{L_{t-1}^l}{\pi_t} = d_\delta + \omega_b \beta^b \mathbb{E}_t \left[\left((1-\delta_t)(L_{t-1}^l)^2 \right) \dot{U}_{t+1}^b \right]$$

The final log-linearisation is,

$$\begin{aligned} & \frac{L_{ss}^l}{\pi_{ss}(\pi_{ss}^b+1)^{\sigma_b}} \left(\hat{L}_{t-1}^l - \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b+1)} \hat{\pi}_t^b - \hat{\pi}_t \right) - d_\delta \\ & = \beta^b \left(\frac{\omega_\delta}{2} \right) (1-\delta_{ss})(L_{ss}^l)^2 (\pi_{ss}^b+1)^{-\sigma_b} \mathbb{E}_t \left[\left(\frac{\delta_{ss}}{1-\delta_{ss}} \right) \hat{\delta}_t + 2\hat{L}_{t-1}^l - \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b+1)} \hat{\pi}_{t+1}^b \right] \end{aligned} \tag{92}$$

A.6 Central Bank

A.6.1 Budget Constraint

The budget constraint of the central bank is,

$$T_t^r - M_t R_t^m = \frac{B_{t-1}^c}{\pi_t} - \frac{B_t^c}{R_t^b}$$

Log-linearisation gives,

$$T_{ss}^r \hat{T}_t^r - R_{ss}^m M_{ss} \left(\hat{R}_t^m + \hat{M}_t \right) = \frac{B_{ss}^c}{\pi_{ss}} \left(\hat{B}_{t-1}^c - \hat{\pi}_t \right) - \frac{B_{ss}^c}{R_{ss}^b} \left(\hat{B}_t^c - \hat{R}_t^b \right) \tag{93}$$

A.6.2 Eligible Assets

Collateralised lending equation is,

$$M_t = \kappa_t \cdot \frac{B_{t-1}}{R_t^m \pi_t}$$

Log-linearisation gives,

$$M_{ss} \hat{M}_t = \kappa_{ss} \frac{B_{ss}}{R_{ss}^m \pi_{ss}} \left(\hat{\kappa}_t + \hat{B}_{t-1} - \hat{\pi}_t - \hat{R}_t^m \right) \tag{94}$$

A.6.3 Feedback Rule

Central bank sets the policy rate according to this feedback ,

$$R_t^m = (R_{t-1}^m)^{\rho_r}(R_{ss}^m)^{1-\rho_r}\left(\frac{\pi_t}{\pi_{ss}}\right)^{\rho_\pi(1-\rho_r)}\left(\frac{Y_t}{Y_{ss}}\right)^{\rho_Y(1-\rho_r)}\left(\frac{Y_t}{Y_{t-1}}\right)^{\rho_d Y(1-\rho_r)} e^{\xi_{R,t}}$$

Log-linearisation gives,

$$\hat{R}_t^m = \rho_r(\hat{R}_{t-1}^m) + (1-\rho_r)\left[\rho_\pi\hat{\pi}_t + \rho_Y\hat{Y}_t + \rho_{dY}(\hat{Y}_t - \hat{Y}_{t-1})\right] + \xi_{R,t} \tag{95}$$

A.7 Government

A.7.1 Government Budget Constraint

The government budget constraint is,

$$G_t + \frac{B_t^g}{R_t^b} = \frac{B_{t-1}^g}{\pi_t} + T_t$$

Log-linearisation gives,

$$G_{ss}\hat{G}_t + \frac{B_{ss}^g}{R_{ss}^b}\left(\hat{B}_t^g - \hat{R}_t^b\right) = \frac{B_{ss}^g}{\pi_{ss}}\left(\hat{B}_{t-1}^g - \hat{\pi}_t\right) + T_{ss}\hat{T}_t \tag{96}$$

A.7.2 Growth Rate of Bonds

The growth rate of bonds is,

$$B_t^g = \Omega B_{t-1}^g$$

Log-linearisation gives,

$$\hat{B}_t^g = \Omega\left(\hat{B}_{t-1}^g - \hat{\pi}_t\right) \tag{97}$$

A.8 Market Clearing

A.8.1 Market Clearing

The market clearing condition is,

$$
\begin{aligned}
Y_t = {} & C_t + G_t + \pi_t^f + \pi_t^b + \pi_t^l + K_t - (1-\varphi)K_{t-1} + \frac{L_t^b}{R_t^c}\left[\Gamma\left(\frac{L_t^b}{L_{t-1}^b}\right)\right] \\
& + \frac{\omega_\delta}{2}\left[(1-\delta_{t-1})L_{t-2}^l\right]^2 + \frac{\omega_\psi}{2}\left[(1-\psi_{t-1})L_{t-2}^b\right]^2 \\
& + \frac{\varrho}{2}\left(\frac{p_t}{(\bar{\pi})^{1-\gamma_p}(\pi_{t-1})^{\gamma_p}p_{t-1}} - 1\right)^2 Y_t
\end{aligned}
$$

Log-linearisation gives,

$$
\begin{aligned}
& Y_{ss}\hat{Y}_t = C_{ss}\hat{C}_t + G_{ss}\hat{G}_t + \pi_{ss}^f\hat{\pi}_t^f + \pi_{ss}^b\hat{\pi}_t^b + \pi_{ss}^l\hat{\pi}_t^l + K_{ss}\hat{K}_t - (1-\varphi)K_{ss}\hat{K}_{t-1} \\
& + (\omega_\delta)(1-\delta_{ss})^2(L_{ss}^l)^2\left[-2\left(\frac{\delta_{ss}}{1-\delta_{ss}}\right)\hat{\delta}_{t-1} + 2\hat{L}_{t-2}^l\right] \\
& + (\omega_\psi)(1-\psi_{ss})^2(L_{ss}^b)^2\left[-2\left(\frac{\psi_{ss}}{1-\psi_{ss}}\right)\hat{\psi}_{t-1} + 2\hat{L}_{t-2}^b\right]
\end{aligned} \tag{98}
$$

A.8.2 Production Function

The aggregate production function is,

$$Y_t = K_t^\alpha N_t^{1-\alpha}$$

Log-linearisation gives,

$$\hat{Y}_t = \alpha\hat{K}_t + (1-\alpha)\hat{N}_t \tag{99}$$

A.8.3 Capital

The aggregated law of motion for capital is,

$$K_t = (1-\varphi)K_{t-1} + \frac{L_t^b}{R_t^c}\left[1 - \Gamma\left(\frac{L_t^b}{L_{t-1}^b}\right)\right]$$

Log-linearisation gives,

$$\hat{K}_t = (1-\varphi)\hat{K}_{t-1} + \frac{L^b_{ss}}{R^c_{ss}K_{ss}}(\hat{L}^b_t - \hat{R}^c_t) \tag{100}$$

A.9 Extra Equations

A.9.1 Household Budget Constraint

Household budget constraint is,

$$\frac{D^l_t}{R^d_t} + C_t - T_t = w_t N_t + \frac{D^l_{t-1}}{\pi_t} - T^r_t$$

Log-linearisation gives us,

$$\begin{aligned}
&\frac{D^l_{ss}}{R^d_{ss}}\left(\hat{D}^l_t - \hat{R}^d_t\right) + C_{ss}(\hat{C}_t) - T_{ss}(\hat{T}_t) \\
&= w_{ss}N_{ss}\left(\hat{w}_t + \hat{N}_t\right) + \frac{D^l_{ss}}{\pi_{ss}}\left(\hat{D}^l_{t-1} - \hat{\pi}_t\right) - T^r_{ss}(\hat{T}^r_t)
\end{aligned} \tag{101}$$

A.9.2 Indexing Rule

The indexing rule is,

$$w_{j,t+1} = (\pi_t)^{\tau_w} w_{j,t}$$

Log-linearisation gives us,

$$\hat{w}_{t+1} - \hat{w}_t = (\pi_{ss})^{\tau_w}(\tau_w \hat{\pi}_t) \tag{102}$$

A.9.3 Balancing equations

There are several budget balancing equations, The log-linearised equations are,

$$\hat{D}_t^l = \hat{L}_t^l \tag{103}$$

$$L_{ss}^l \hat{L}_t^l = M_{ss}^p \hat{M}_t^p + B_{ss} \hat{B}_t + L_{ss}^b \hat{L}_t^b \tag{104}$$

$$\hat{M}_t^p = \hat{B}_t^c \tag{105}$$

$$B_{ss}^g \hat{B}_t^g = B_{ss}^c \hat{B}_t^c + B_{ss} \hat{B}_t \tag{106}$$

A.9.4 Shocks

Besides the shock on the feedback rule, the shock I impose on κ_t is similar to that of Hilberg and Hollmayr (2011). The equation for κ is defined as $\kappa_t = \rho_\kappa \kappa_{t-1} \xi_{\kappa,t}$, where is $\xi_{\kappa,t}$ is the innovation. Log-linearisation gives,

$$\hat{\kappa}_t = \rho_\kappa \hat{\kappa}_{t-1} + \xi_{\kappa,t} \tag{107}$$

Another plausible way to impose a shock on liquidity, as in Niestroj et al. (2013), is to simply add it in this equation, as follows,

$$M_t \leq \kappa_t \cdot \frac{B_{t-1}}{R_t^m \pi_t} + \xi_{M,t} \tag{108}$$

This method was also attempted, but the haircut method was favored, as it framed the question more accurately.

B Calibration and Steady States

In the following section there is a brief discussion on the implied steady state values for this model. The value of the β parameter is set by imposing a value for the deposit rate, $R^d = 1.006$, and inflation, $\pi = 1.0051$, in steady state. From the household's first order conditions one gets, $\beta = \pi_{ss}/R^d_{ss}$. The discount factor is structured to be the same for all sectors, as in de Walque et al. (2010). Discussion on the interest rate transmission in provided in the chapter. The value of the multiplier on the household budget constraint, with habit formation set at $h = 0.57$ and coefficient of relative risk aversion at $\sigma^c = 1.35$, is,

$$\lambda^h_{ss} = h(C_{ss} - hC_{ss})^{-\sigma_c}$$

In the instance above the value for C_{ss} is found by imposing a steady state consumption to output ratio. The Calvo parameter on wages is calibrated to be $\tau_w = 0.62$ and the elasticity of substitution between labour varieties is $\eta = 0.2$, which means I can deliver the value for $\pi^{w^*}_{ss}$, which is,

$$\pi^{w^*}_{ss} = \frac{1 - \theta_w \pi_{ss}^{(1-\eta)(\tau_w - 1)}}{(1-\theta_w)^{\frac{1}{1-\eta}}}$$

With this value established, the following value for f_{ss} can be found, given the inverse Frisch elasticity of labour supply, $\sigma_n = 2.4$, and the wage indexation parameter, $\tau^w = 0.62$. The equation is as follows,

$$f_{ss} = \frac{(\pi^{w^*}_{ss})^{-\eta(1+\sigma_n)}(N_{ss})^{(1+\sigma_n)}}{1 - \beta\theta_w(\pi_{ss})^{\eta(1-\tau_w)(1+\sigma_n)}}$$

With the value of f_{ss} determined and $N_{ss} = 0.33$, I have that,

$$w^*_{ss} = \frac{1 - \beta\theta_w f_{ss}(\pi_{ss})^{(1-\eta)(\tau_w - 1)}}{\frac{\eta-1}{\eta}\lambda^h_{ss}(\pi^{w^*}_{ss})^{\eta} N_{ss}}$$

The value for output is normalised to one, while the labour steady state value is $N_{ss} = 0.33$. With the capital share of output calibrated as, $\alpha = 0.3$, the implied steady state value of K_{ss} is,

$$K_{ss} = \left(\frac{Y_{ss}}{N_{ss}^{(1-\alpha)}}\right)^{\frac{1}{\alpha}}$$

Given this value of K_{ss} and the depreciation rate of, $\varphi = 0.03$, the steady state of loans extended to firms is defined as $L^b_{ss} = K_{ss}R^c_{ss}\varphi$. Next, the steady state wage is given by

$w_{ss} = (1-\alpha)(K_{ss}^{\alpha})(N_{ss}^{-\alpha})$. With these values I can determine the multiplier for the firm budget constraint as,

$$\lambda_{ss}^{f} = \frac{\alpha(K_{ss})^{\alpha-1}(N_{ss})^{1-\alpha}}{1-\beta(1-\varphi)}$$

The value of the marginal cost in steady state is given by $mc_{ss} = (\epsilon - 1)/\epsilon$, where $\epsilon = 3$. Having normalised merchant bank profit in steady state to $\pi_{ss}^{b} = 0.0001$, the value of the multiplier on the collateralised lending constraint is,

$$\eta_{ss}^{m} = (\pi_{ss}^{b} + 1)^{-\sigma_b}\left(\frac{1}{R_{ss}^{m}} - 1\right)$$

The multiplier on the balanced budget condition is written as,

$$\Upsilon_{ss} = (\pi_{ss}^{b} + 1)^{-\sigma_b}\left(\frac{\beta^{b}}{\pi_{ss}} - 1\right)$$

The steady state for the bond rate is endogenously determined by,

$$R_{ss}^{b} = \frac{\pi_{ss}}{\beta^{b}} \cdot \left[\frac{(\pi_{ss}^{b}+1)^{-\sigma_b}}{(\pi_{ss}^{b}+1)^{-\sigma_b} + \kappa_{ss}\eta_{ss}^{m}}\right] - \frac{\Upsilon_{ss}}{(\pi_{ss}^{b}+1)^{-\sigma_b}}$$

The repayment rate of interbank loans is calibrated to be $\delta = 0.995$, which means the utility cost from merchant bank default is,

$$\omega_{\delta} = \frac{\left(\frac{1}{R_{ss}^{l}} - \frac{\beta\delta_{ss}}{\pi_{ss}}\right)}{\beta^{2}L_{ss}^{l}(1-\delta_{ss})^{2}}$$

The repayment rate of firm loans is set at $\psi = 0.975$, which combined with the revealed value of ω_{δ}, gives,

$$d_{\delta} = \frac{L_{ss}^{l}}{\pi_{ss}} - \frac{\left(\frac{\omega_{\delta}}{2}\right)\beta(1-\delta_{ss})(L_{ss}^{l})^{2}}{(\pi_{ss}^{b}+1)^{\sigma_b}}$$

In addition, with ω_{δ} determined, one can back out the value of ω_{ψ} from the market clearing condition, as follows,

$$\omega_{\psi} = \frac{(Y_{ss} - C_{ss} - G_{ss} - \pi_{ss}^{f} - \pi_{ss}^{l} - \pi_{ss}^{b} - \varphi K_{ss} - \omega_{\delta})}{(1-\psi_{ss}(L_{ss}^{b})^{2})\left[(1-\delta_{ss})(L_{ss}^{l})^{2}\right]^{-1}}$$

In the above equation the values for G_{ss}, π^f_{ss} and π^l_{ss} are determined from steady state ratios imposed, and similar to de Walque et al. (2010), $L^l_{ss} = 0.7L^b_{ss}$. Given ω_ψ, the equation for d_ψ is,

$$d_\psi = \frac{L^b_{ss}}{\pi_{ss}} - \beta\left(\frac{\omega_\psi}{2}\right)(1-\psi_{ss})L^b_{ss}$$

With T_{ss} given by the imposed steady state ratios, and $D^l_{ss} = L^L_{ss}$ from the deposit bank balanced budget constraint, remittances to households is given by,

$$T^r_{ss} = -\left(\frac{D^l_{ss}}{R^d_{ss}} + C_{ss} + T_{ss} - w_{ss}N_{ss} - \frac{D^l_{ss}}{\pi_{ss}}\right)$$

Total bond supply is reflected in the following equation,

$$B^T_{ss} = (T_{ss} - G_{ss})\left[\frac{1}{R^b_{ss}} - \frac{1}{\pi_{ss}}\right]^{-1}$$

This allows me to write, $B^c_{ss} = B^g_{ss} - B_{ss}$, which gives $M^p_{ss} = B^c_{ss}$ from the initial condition. Finally, the reserves steady state is $M_{ss} = \kappa B_{ss}(R^m_{ss}\pi_{ss})^{-1}$.

C Reserve Requirement

One of the potential shortcomings mentioned in the thesis is that there is no explicit role for money. In this largely cashless economy it could be useful to introduce some cash-in-advance constraints on the household, deposit and merchant banks. These constraints were initially attempted, but introduced a much greater deal of complexity, somewhat detracting from the central message. Consider the merchant bank for a moment. A CIA constraint on this bank can be motivated as a minimum reserves requirement. Below is a representation of what such requirement might look like.

In particular, it would require the merchant bank to hold a certain fraction of its interbank loans in the form of liquidity, generating demand for M^p_t. Normally with this type of constraint the commercial banks is forced to hold a fraction of deposits, but the merchant bank does not have access to household deposits. The minimum reserve requirement takes the following form,

$$\Theta L^l_{t-1} \leq M^p_t$$

with Θ the fraction of interbank loans held. The first order conditions of the merchant bank will be altered in the following way,

$$
\begin{aligned}
&\dot{U}_t^b = R_t^m(\dot{U}_t^b + \eta_t) \\
&\frac{1}{R_t^b} = 1 + (\dot{U}_t^b)^{-1}\left[\beta^b \mathbb{E}_t\left(\frac{\kappa \cdot \eta_{t+1}}{\pi_{t+1}}\right) + \zeta_t\right] \\
&\dot{U}_t^b\left(\frac{1}{R_t^l} + \frac{1}{R_t^c}\right) = \beta^b \mathbb{E}_t\left[\left(\frac{\delta_{t+1} + \Theta \cdot \zeta_{t+1} + \psi_{t+1}}{\pi_{t+1}}\right)\dot{U}_{t+1}^b\right] + \\
&(\beta^b)^2 \mathbb{E}_{t+1}\left[\left(\omega_\delta(1-\delta_{t+1})^2 L_t^l\right)\dot{U}_{t+2}^b\right] \\
&\dot{U}_t^b \frac{L_{t-1}^l}{\pi_t} = d_\delta + \omega_\delta \beta^b \mathbb{E}_t\left[\left((1-\delta_t)(L_{t-1}^l)^2\right)\dot{U}_{t+1}^b\right]
\end{aligned}
$$

where ζ is the newly introduced multiplier on the reserve requirement. There are several differences visible when incorporating these changes. For example it can be already be seen with the partial equilibrium analysis that the relationship between deposit, credit, interbank lending and bond rates are different, becoming more complex to analyse in levels,

$$R_{ss}^d = R_{ss}^l \delta_{ss} = R_{ss}^c(\Theta\zeta_{ss} + \psi_{ss}) = R_{ss}^b \cdot \left((1 - R_{ss}^b - \zeta_{ss} \cdot R_{ss}^b) \cdot \kappa_{ss}\eta_{ss}\right)^{-1}$$

While it is worthwhile to introduce this constraint, the question has to be asked, does it alter the result in a significant way? In general, the results obtained are highly similar. Does it add to the understanding of the model? In this instance, it might provide additional motivation as to the merchant bank's demand for money, but on the other hand it makes the model more complex and difficult to navigate. For the sake of brevity, and clarity, it was excluded.

Chapter 7

D Log-Linearised Model

The household, firms and deposit banks are the same as in the previous chapter. This section only represents the log-linearised equations that are introduced in this chapter. Namely those that are related to the introduction of long-term bonds.

D.1 Merchant Bank

D.1.1 Long-Term Bonds

The first order condition with respect to long-term bonds is,

$$(\partial B_t^L) \qquad \dot{U}_t^b p_t^L = \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b + \kappa^l \cdot \eta_{t+1} p_{t+1}^L R_{t+1}^L}{\pi_{t+1}} \right) - \Upsilon_t \mathbb{E}_t \left(p_{t+1}^L R_{t+1}^L \right)$$

Eliminating Υ_t, this equation can be rewritten as,

$$\begin{aligned} \dot{U}_t^b p_t^L = {} & \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b + \kappa^l \cdot \eta_{t+1} p_{t+1}^L R_{t+1}^L}{\pi_{t+1}} \right) \\ & - \beta^b \mathbb{E}_t \left(\frac{\dot{U}_{t+1}^b p_{t+1}^L R_{t+1}^L}{\pi_{t+1}} \right) + \dot{U}_t^b \mathbb{E}_t \left(p_{t+1}^L R_{t+1}^L \right) \end{aligned}$$

Which can then be simplified to,

$$\dot{U}_t^b p_t^L = \frac{\beta^b}{\pi_{t+1}} \mathbb{E}_t \left(\dot{U}_{t+1}^b + \kappa^l \cdot \eta_{t+1} p_{t+1}^L R_{t+1}^L - \dot{U}_{t+1}^b p_{t+1}^L R_{t+1}^L \right) + \dot{U}_t^b \mathbb{E}_t \left(p_{t+1}^L R_{t+1}^L \right)$$

The final log-linearisation is,

$$\begin{aligned} & \left(\frac{1}{R_{ss}^L - \Phi} \right) (\pi_{ss}^b + 1)^{-\sigma_b} \left[- \left(\frac{R_{ss}^L}{R_{ss}^L - \Phi} \right) \hat{R}_t^L - \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_t^b \right] \\ & = \frac{\beta^b}{\pi_{ss}(\pi_{ss}^b + 1)^{\sigma_b}} \left[-\hat{\pi}_{t+1} - \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_{t+1}^b \right] + \frac{\beta^b \kappa_{ss}^l \eta_{ss}^m R_{ss}^L}{(R_{ss}^L - \Phi)\pi_{ss}} \left[\hat{\kappa}_{t+1}^l - \hat{\pi}_{t+1} + \eta_{t+1}^{\hat{m}} - \left(\frac{\Phi}{R_{ss}^L - \Phi} \right) \hat{R}_t^L \right] \\ & - \frac{\beta^b R_{ss}^L}{(R_{ss}^L - \Phi)\pi_{ss}(\pi_{ss}^b + 1)^{\sigma_b}} \left[- \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_{t+1}^b - \hat{\pi}_{t+1} - \left(\frac{\Phi}{R_{ss}^L - \Phi} \right) \hat{R}_{t+1}^L \right] \\ & + \frac{R_{ss}^L}{(R_{ss}^L - \Phi)(\pi_{ss}^b + 1)^{\sigma_b}} \left[- \frac{\pi_{ss}^b \sigma_b}{(\pi_{ss}^b + 1)} \hat{\pi}_{t+1}^b - \left(\frac{\Phi}{R_{ss}^L - \Phi} \right) \hat{R}_{t+1}^L \right] \end{aligned} \tag{109}$$

D.2 Government

D.2.1 Government Budget Constraint

The government budget constraint is,

$$G_t + \frac{B_{t-1}^{TS}}{\pi_t} + \frac{p_t^L R_t^L B_{t-1}^{TL}}{\pi_t} = \frac{B_t^{TS}}{R_t^b} + p_t^L B_t^{TL} + T_t$$

The final log-linearisation is,

$$G_{ss}\hat{G}_t + \frac{B^{TS}_{ss}}{\pi_{ss}}(\hat{B}^{TS}_{t-1} - \hat{\pi}_t) + \frac{B^{TL}_{ss}R^L_{ss}}{(R^L_{ss} - \Phi)\pi_{ss}}\left[\hat{B}^{TL}_{t-1} - \hat{\pi}_t - \left(\frac{\Phi}{R^L_{ss} - \Phi}\right)\hat{R}^L_t\right]$$
$$= \frac{B^{TS}_{ss}}{R^b_{ss}}(\hat{B}^{TS}_t - \hat{R}^b_t) + T_{ss}\hat{T}_t + \frac{B^{TL}_{ss}}{(R^L_{ss} - \Phi)}\left[\hat{B}^{TL}_t - \left(\frac{R^L_{ss}}{R^L_{ss} - \Phi}\right)\hat{R}^L_t\right] \tag{110}$$

D.2.2 Growth Rate of Long-Term Bonds

The equation for the growth of long-term bonds is,

$$p^L_t B^L_t = \left(\frac{p^L_{t-1}B^L_{t-1}}{\pi_t}\right)^{\rho_b} e^{\xi^L_t}$$

The final log-linearisation is,

$$\hat{B}^L_t = \left(\frac{R^L_{ss}}{R^L_{ss} - \Phi}\right)\hat{R}^L_t + \rho_b\left[\hat{B}^L_{t-1} - \left(\frac{R^L_{ss}}{R^L_{ss} - \Phi}\right)\hat{R}^L_{t-1} - \hat{\pi}_t\right] + \xi^L_t$$

D.3 Central Bank

D.3.1 Budget Constraint

The first order condition with respect to bonds is,

$$T^r_t - \frac{B^{CS}_{t-1}}{\pi_t} + \frac{B^{CS}_t}{R^b_t} = \frac{p^L_t R^L_t B^{CL}_{t-1}}{\pi_t} - p^L_t B^{CL}_t + \left(M^p_t - \frac{M^p_{t-1}}{\pi_t}\right)R^m_t$$

The final log-linearisation is,

$$T^r_{ss}\hat{T}^r_t - \frac{B^{CS}_{ss}}{\pi_{ss}}(\hat{B}^{CS}_{t-1} - \hat{\pi}_t) + \frac{B^{CS}_{ss}}{R^b_{ss}}(\hat{B}^{CS}_t - \hat{R}^b_t) + \frac{B^{CL}_{ss}}{(R^L_{ss} - \Phi)}\left[\hat{B}^{CL}_t - \left(\frac{R^L_{ss}}{R^L_{ss} - \Phi}\right)\hat{R}^L_t\right]$$
$$= \frac{B^{CL}_{ss}R^L_{ss}}{(R^L_{ss} - \Phi)\pi_{ss}}\left[\hat{B}^{CL}_{t-1} - \hat{\pi}_t - \left(\frac{\Phi}{R^L_{ss} - \Phi}\right)\hat{R}^L_t\right] + M^p_{ss}R^m_{ss}\left(\hat{M}^p_t + \hat{R}^m_t\right)$$
$$- M^p_{ss}R^m_{ss}(\pi_{ss})^{-1}(\hat{M}^p_{t-1} + \hat{R}^m_t - \hat{\pi}_t) \tag{111}$$

D.3.2 Eligible Assets

The first order condition with respect to bonds is,

$$M_t = \kappa_t^s \cdot \frac{B_{t-1}^S}{R_t^m \pi_t} + \kappa_t^l \cdot \frac{p_t^L R_t^L B_{t-1}^L}{R_t^m \pi_t}$$

The final log-linearisation is,

$$\begin{aligned} M_{ss}\hat{M}_t &= \kappa_{ss}^s \frac{B_{ss}}{R_{ss}^m \pi_{ss}} \left(\hat{\kappa}_t^s + \hat{B}_{t-1} - \hat{\pi}_t - \hat{R}_t^m\right) \\ &+ \frac{\kappa_{ss}^l B_{ss}^L R_{ss}^L}{(R_{ss}^L - \Phi)\pi_{ss} R_{ss}^m} \left[\hat{\kappa}_t^l + \hat{B}_{t-1}^l - \left(\frac{\Phi}{R_{ss}^L - \Phi}\right) \hat{R}_t^L - \hat{\pi}_t - \hat{R}_t^m\right] \end{aligned} \tag{112}$$

D.4 Extra Equations

D.4.1 Balanced Budget Constraints

There are three affected balanced budget conditions.

$$\begin{aligned} L_{ss}^l \hat{L}_t^l &= M_{ss}^p \hat{M}_t^p + L_{ss}^b \hat{L}_t^b + B_{ss}^S B_t^S + \frac{B_{ss}^{CL} R_{ss}^L}{(R_{ss}^L - \Phi)\pi_{ss}} \left[\hat{B}_t^{CL} - \hat{\pi}_{t+1} - \left(\frac{\Phi}{R_{ss}^L - \Phi}\right) \hat{R}_{t+1}^L\right] \\ M_{ss}^p \hat{M}_t^p &= B_{ss}^{CS} \hat{B}_t^{CS} + \frac{B_{ss}^{CL}}{(R_{ss}^L - \Phi)} \left[\hat{B}_t^{CL} - \left(\frac{R_{ss}^L}{R_{ss}^L - \Phi}\right) \hat{R}_t^L\right] \\ B_{ss}^{TL} \hat{B}_t^{TL} &= B_{ss}^L \hat{B}_t^L + B_{ss}^{CL} \hat{B}_t^{CL} \end{aligned}$$

E Calibration and Steady States

Calibration for the household, firm and deposit bank remain the same. However, with respect to the merchant bank, central bank and government there are a few changes. The determination of the value of Φ requires setting the R_{ss}^L. I follow the work of Chen et al. (2012) in this regard, which delivers,

$$\Phi = (5.5 \cdot R_{ss}^L - R_{ss}^L)/5.5$$

The next change observed is with the merchant bank. With equations becoming unwieldy as this point, I had to break them into parts. Presented below are some of the selected substitutions made, arising from the the equations in the model. These equations are named, X_s, X_l, Z_s and Z_l. The values of X_s and X_l come from a combination of the collateralised borrowing constraint and the central bank budget constraint, while Z_s and Z_l come from the addition of the balanced budget condition for the merchant bank.

$$\begin{aligned}
X_s &= \frac{\pi_{ss}\kappa^s_{ss}}{\pi_{ss} - 1(R^m_{ss}\pi_{ss})} + 1 \\
X_l &= \frac{\pi_{ss}}{(\pi_{ss} - 1)}\left(\frac{\kappa^l_{ss}R^L_{ss}}{(R^L_{ss} - \Phi)\pi_{ss}R^m_{ss}} + \frac{1}{(R^L_{ss} - \Phi)}\right) \\
Z_s &= \left(\frac{1}{\pi_{ss}} - \frac{1}{R^b_{ss}} - \frac{\kappa^s_{ss}}{(R^m_{ss}\pi_{ss})}\right) \\
Z_l &= \left(\frac{R^L_{ss}}{(R^L_{ss} - \Phi)\pi_{ss}} - \frac{1}{(R^L_{ss} - \Phi)} - \frac{\kappa^l_{ss}R^L_{ss}}{(R^L_{ss} - \Phi)\pi_{ss}R^m_{ss}}\right)
\end{aligned}$$

With these components, I can now show how the rest of the implied values are achieved. First, the value for B^S_{ss} can be given by,

$$B^S_{ss} = \left[\frac{L^l_{ss}}{R^l_{ss}} - \frac{\delta_{ss}L^l_{ss}}{(\pi_{ss})} + \frac{\psi_{ss}L^b_{ss}}{\pi_{ss}} - \frac{L^b_{ss}}{R^c_{ss}} - \pi^b_{ss} - \frac{\omega_\delta}{2}\left((1 - \delta_{ss})L^l_{ss}\right)^2\right] \cdot (Z_s X_l / X_s + Z_l)^{-1}$$

With the value for B^S_{ss} determined, I can now establish the value of B^L_{ss} with the following equation,

$$B^L_{ss} = \frac{L^l_{ss}}{X_l} - (X_s/X_l)B^S_{ss} - \frac{L^b_{ss}}{X_l}$$

Using the collateralised lending equation, I can determine the value for M_{ss}, as,

$$M_{ss} = \frac{\kappa^s_{ss}B^S_{ss}}{R^m_{ss}\pi_{ss}} + \frac{\kappa^l_{ss}B^l_{ss}R^L_{ss}}{(R^L_{ss} - \Phi)\pi_{ss}R^m_{ss}}$$

With the value of M_{ss} identified, the value for M^p_{ss} is given by,

$$M^p_{ss} = \frac{M_{ss}\pi_{ss}}{(\pi_{ss} - 1)}$$

Once again, to avoid messy equations, I define a component of the equation for B^{CL}_{ss} as Y_l,

$$Y_l = \frac{1}{(R^L_{ss} - \Phi)}\left(\frac{1}{\pi_{ss}} - \frac{1}{R^b_{ss}}\right) + \frac{R^L_{ss}}{\pi_{ss}(R^L_{ss} - \Phi)}$$

The equation for B^{CL}_{ss} can then be written as,

$$B^{CL}_{ss} = \frac{M_{ss}R^m_{ss}}{Y_l} - T^r_{ss}$$

Table 1: Calibrated parameters

Parameter	Description	Value
h	Habit formation (consumption)	0.57
σ^c	Coefficient of relative risk aversion	1.35
σ^n	Inverse Frisch elasticity of labour supply	2.4
σ^l	Coefficient for deposit bank (NB)	1.35
σ^b	Coefficient for merchant bank (NB)	1.35
η	Elasticity of substitution between labor varieties	3
ϵ	Elasticity of substitution between goods varieties	3
τ^w	Wage indexation parameter	0.62
θ^w	Calvo parameter (wages)	0.2
α	Capital share of output	0.3
φ	Depreciation rate	0.03
θ	Firms' investment adjustment cost	6.77
ϱ	Price adjustment cost – Calvo parameter (prices)	120
γ_p	Another component of wage adjustment cost	0.47
Γ	Bond supply growth rate	1.055
ρ_r	Interest rate smoothing coefficient (Taylor Rule)	0.5
ρ_π	Feedback coefficient to inflation in monetary policy rule	1.68
ρ_y	Feedback coefficient to output growth deviation	0.01
ρ_{d^y}	Feedback coefficient to output growth deviation	0.16

With B_{ss}^{CL}, the equation used for B_{ss}^{CS} is,

$$B_{ss}^{CS} = M_{ss}^{p} - \frac{B_{ss}^{CL}}{(RL_s s - \Phi)}$$

Finally, the values for B_{ss}^{CS} and B_{ss}^{CS} are determined form the balanced budget conditions for short- an long-term bonds, which are given by,

$$B_{ss}^{TL} = B_{ss}^{L} + B_{ss}^{CL}$$
$$B_{ss}^{TS} = B_{ss}^{S} + B_{ss}^{CS}$$

This concludes the discussion on implied steady state values for Chapter 7.

Table 2: Imposed steady states and ratios

Parameter	Description	Value
π	Inflation	1.051
R^d	Deposit rate	1.065
R^m	Policy rate (Central Bank)	1.06
R^L	Long-term bond rate	1.075
δ	Repayment rate (deposit bank)	0.995
ψ	Repayment rate (merchant bank)	0.975
N	Labour steady state	0.33
C/Y	Consumption spending to output ratio	0.42
π^f/Y	Firm profit to output ratio	0.1
π^l/Y	Deposit bank profit to output ratio	0.0001
π^b/Y	Merchant bank profit to output ratio	0.0001
G/Y	Government spending to output ratio	0.1854
T/Y	Taxation to output ratio	0.187
L^l/L^b	Interbank to firm loan ratio	0.65

F Code

This section includes the code used for the final model with long-term bonds. It depicts the contractionary interest rate shock from Chapter 7.

```
//=====================================================================
// FINAL MODEL (Contractionary Interest Rate Policy)
//=====================================================================

//=====================================================================
// ENDOGENOUS VARIABLES
//=====================================================================

var c_hat               // consumption
    lambdah_hat         // Lagrange multiplier (household)
    Rd_hat              // nominal (interest) deposit rate
    PI_hat              // inflation
    f_hat               // variable for recursive formulation of wage setting
    w_hat               // real wage
    wstar_hat           // optimal real wage
    N_hat               // aggregate labour demand
    K_hat               // capital
    lambdaf_hat         // Lagrange multiplier (firm)
    mc_hat              // marginal cost
    Rc_hat              // credit rate (rate at which loans are provided)
    Lb_hat              // loans to firms
    psi_hat             // default rate on loan repayment for firms is (1-psi)
    Rl_hat              // interbank rate, rate at which loans are provided to merchant banks
    deltta_hat          // default rate on loan repayment for merchant banks is (1-delta)
    Ll_hat              // loans to merchant bank (interbank loans)
    Rm_hat              // refinancing rate (policy rate)
    t_hat               // tax
    y_hat               // aggregate output
    Dl_hat              // deposit holdings
    pil_hat             // deposit bank profit
    pif_hat             // firm profit
    pib_hat             // merchant bank profit
    tr_hat              // central bank transfers
    Rb_hat              // bond rate
    I_hat               // newly issued reserves
    Bs_hat              // merchant bank short-term bond holdings
    Bts_hat             // total short-term bond supply
    Bcs_hat             // central bank short-term bond holdings
    eeta_hat            // multiplier on collateral constraint
    g_hat               // government spending
    RL_hat              // yield to maturity
    Bcl_hat             // central bank long-term bond holdings
    Btl_hat             // total supply of long-term bonds
    Bl_hat              // merchant bank long-term bond holdings
    mp_hat;             // outright purchases of reserves
```

```
//======================================================================
// EXOGENOUS VARIABLES
//======================================================================

varexo em elb emp ekp;

//======================================================================
// PARAMETERS
//======================================================================

parameters  h           // consumption habit formation
//          betta_h     // discount factor (household)
//          betta_f     // discount factor (firm)
//          betta_l     // discount factor (deposit bank)
//          betta_b     // discount factor (merchant bank)
            sigmac      // coefficient of relative risk aversion (consumption)
            sigman      // inverse Frisch elasticity of labour supply
            eta         // elasticity of substitution between labor varieties
            epsilon     // elasticity of substitution between goods varieties
            tauw        // wage indexation parameter
            thetaw      // Calvo parameter wages (portion that can no change their wage)
            alppha      // capital share of output
//          dpsi        // disutility associated with default (firm)
            varphi      // depreciation rate
            theta       // investment adjustment cost (firm)
//          wpsi        // pecuniary cost of default (firm)
            varrho      // price adjustment cost (firm) (Calvo parameter for prices)
            gammap      // wage adjustment cost
//          ddelta      // disutility from default (merchant bank)
//          wdelta      // pecuniary cost from default (merchant bank)
            varpi       // bond supply growth rate
            kappal      // haircut (long-term bonds - composition)
            kappas      // haircut (short-term - size)
            rhoR        // interest smoothing coefficient Taylor rule
            rhoPI       // feedback coefficient to inflatioon monetary policy rule
            rhoy        // feedback coefficient to output growth deviation in monetary policy rule
            rhody       // similar to the previus one (think on this one a bit)
            vartheta    // fraction of repos, set exogenously by the central bank
            mu          // minimum reserve ratio
            Phii        // paramater to solve indeterminancy
            sigmal      // coefficient for deposit bank
            sigmab      // coefficient for merchant bank
            gaama       // bond growth rate
            chii        // paramater to solve indeterminancy
```

```
//=========================================================================
// STEADY STATES AND RATIOS (IMPOSED)
//=========================================================================

            PI_ss
            Rd_ss
            Rm_ss
            psi_ss
            deltta_ss
            N_ss
            y_ss
            gy_ss
            cy_ss
            pify_ss
            pily_ss
            piby_ss
            ty_ss
            rrho_ss
            RL_ss
            rhob

//=========================================================================
// SHOCKS
//=========================================================================

            std_m
            std_lb
            std_mp;

//=========================================================================
// CALIBRATED PARAMETER VALUES (IMPOSED)
//=========================================================================

h               = 0.57;               // DW DSGE. Alternative is 0.55 from CS or 0.97 from FV.
sigmac          = 1.35;               // DW DSGE
sigman          = 2.4;                // DW DSGE. Alternative is 2 from CS or 1.17 from FV.
sigmal          = 1.35;               // DW DSGE
sigmab          = 1.35;               // DW DSGE
eta             = 3;                  // CS. Alternative is 3 from DW DSGE or 10 from FV.
epsilon         = 3;                  // CS. Alternative is 3 from DW DSGE or 10 from FV.
tauw            = 0.62;               // FV. This is an estimated parameter, form the data.
thetaw          = 0.2;                // FV. Estimated parameter.
alppha          = 0.33;               // CS. This is the same as DW. However, FV estimated 0.22.
varphi          = 0.025;              // DW DSGE. FV has it as 0.025, CS as 0.03, SW as 0.025.
theta           = 6.77;               // DW DSGE.
varrho          = 260;                // DW DSGE. Alternative is 120 from Niestroj.
gammap          = 0.47;               // DW DSGE.
varpi           = 2;                  // Niestroj.
kappas          = 0.9;                // This is the haircut on short-term bonds
kappal          = 0.1;                // Haircut on long-term bonds
rhoR            = 0.7;                // CS. Parameter estimate. The prior chosen is 0.7.
rhoPI           = 1.68;               // CS. Parameter estimate. Prior chosen is 1.5.
rhoy            = 0.01;               // CS. Parameter estimate. Prior chosen is 0.01.
rhody           = 0.16;               // DW DSGE. Alternate in Smets and Wouters.
vartheta        = 0.1;                // CS. Fraction of money held outright.
```

```
mu              = 0.02;               // CS.
Phii            = 0;
gaama           = 1.055;
chii            = 0.001;
rhob            = 0.9;

//==========================================================================
// STEADY STATES (IMPOSED)
//==========================================================================
rrho_ss         = 1.025;
PI_ss           = 1.051;
Rd_ss           = 1.07;
Rm_ss           = 1.06;
RL_ss           = 1.075;
psi_ss          = 0.975;
deltta_ss       = 0.995;
N_ss            = 0.3;
y_ss            = 1;


//==========================================================================
// CALIBRATED RATIOS (IMPOSED)
//==========================================================================
gy_ss           = 0.1854;               // This is the government spending to gdp ratio
cy_ss           = 0.45;                 // Consumption to gdp ratio
pify_ss         = 0.1;                  // Firm profit to gdp ratio
pily_ss         = 0.0001;               // Deposit bank profit to gdp ratio
piby_ss         = 0.0001;               // Merchant bank to gdp ratio
ty_ss           = 0.18;


//==========================================================================
// SHOCK PARAMETERS
//==========================================================================

std_m           = 1;
std_lb          = 10;
std_mp          = 10;

//==========================================================================

model(linear);

//==========================================================================
// STEADY STATES (IMPLIED)
//==========================================================================

// Steady states as functions of parameters and calibrated values


# Rc_ss         = Rd_ss/psi_ss;
# Rl_ss         = Rd_ss/deltta_ss;
# PSI           = (5.5*RL_ss - RL_ss)/5.5;
# mc_ss         = (epsilon - 1)/epsilon;
# betta_h       = PI_ss/(Rd_ss);
# betta_f       = betta_h;
```

```
# betta_l       = PI_ss/(Rl_ss*deltta_ss);
# betta_b       = PI_ss/(Rc_ss*psi_ss);
# PIstarw_ss    = ((1 - thetaw*(PI_ss)^((1 - eta)*(tauw - 1)))/((1 - thetaw))^(1/(1 - eta)));
# f_ss          = (((PIstarw_ss)^(-eta*(1 + sigman)))*(N_ss)^(1 + sigman))/
                  (1 - betta_h*thetaw*(PI_ss)^(eta*(1 - tauw)*(1 + sigman)));
# K_ss          = (y_ss/(N_ss^(1-alppha)))^(1/alppha);
# Lb_ss         = K_ss*Rc_ss*varphi;
# ky_ss         = K_ss/y_ss;
# g_ss          = gy_ss*y_ss;
# c_ss          = cy_ss*y_ss;
# pif_ss        = pify_ss*y_ss;
# pil_ss        = pily_ss*y_ss;
# pib_ss        = piby_ss*y_ss;
# eeta_ss       = ((pib_ss+1)^(-sigmab))*(1/(Rm_ss)-1);
# Rb_ss         = Rd_ss*((pib_ss+1)^(-sigmab)/(eeta_ss*kappas + ((pib_ss+1)^(-sigmab))));
# Dl_ss         = 0.7*Lb_ss;
# Ll_ss         = Dl_ss;
# w_ss          = (1-alppha)*(K_ss^(alppha))*(N_ss^(-alppha));
# t_ss          = ty_ss*y_ss;
# lambdaf_ss    = (alppha*((K_ss)^(alppha-1))*((N_ss)^(1-alppha)))/(1-(betta_f*(1-varphi)));
# lambdah_ss    = h*((c_ss-h*c_ss)^(-sigmac));
# wstar_ss      = (1 - betta_h*thetaw*(PI_ss^((1-eta)*(tauw-1)))*f_ss)/
                  ((eta-1/eta)*(lambdah_ss)*(PIstarw_ss^(-eta))*N_ss);
# wdelta        = ((1/Rl_ss) - ((betta_b*(deltta_ss))/PI_ss))/((betta_b^2)*Ll_ss*((1-deltta_ss)^2));
# wpsi          = ((y_ss - c_ss - g_ss - pif_ss - pib_ss - pil_ss - varphi*K_ss - (wdelta)*((1-deltta_ss)*Ll_ss)^2)/
                  (((1-psi_ss)*Lb_ss)^2));
# dpsi          = (Lb_ss/PI_ss) - (betta_f*((wpsi/2)*(1-psi_ss))*Lb_ss^2);
# ddelta        = (((Ll_ss/PI_ss) - (wdelta/2)*betta_b*(1-deltta_ss)*Ll_ss^2))/((pib_ss+1)^(sigmab));
# tr_ss         = -(Dl_ss/Rd_ss + c_ss + t_ss - w_ss*N_ss - Dl_ss/PI_ss);
# Xs            = (PI_ss/(PI_ss-1))*(kappas/(Rm_ss*PI_ss)) + 1;
# Xl            = (PI_ss/(PI_ss-1))*(kappal*RL_ss/((RL_ss - PSI)*PI_ss*Rm_ss)) + (1/(RL_ss - PSI));
# Zs            = (1/PI_ss - 1/Rb_ss - kappas/(Rm_ss*PI_ss));
# Zl            = (RL_ss/((RL_ss - PSI)*PI_ss) - 1/(RL_ss - PSI) - kappal*RL_ss/((RL_ss - PSI)*PI_ss*Rm_ss));
# Bs_ss         = (Ll_ss/Rl_ss - deltta_ss*Ll_ss/(PI_ss) + psi_ss*Lb_ss/PI_ss -
                  Lb_ss/Rc_ss - pib_ss - (wdelta/2)*((1-deltta_ss)*Ll_ss)^2)/((Zs*Xl)/Xs + Zl);
# Bl_ss         = (Ll_ss/Xl - (Xs/Xl)*Bs_ss - Lb_ss/Xl);
# I_ss          = ((kappas*Bs_ss)/(Rm_ss*PI_ss) + kappal*Bl_ss*RL_ss/((RL_ss - PSI)*PI_ss*Rm_ss));
# mp_ss         = (I_ss*PI_ss/(PI_ss-1));
# Yl            = ((1/(RL_ss - PSI))*((1/PI_ss) - (1/Rb_ss)) + RL_ss/((PI_ss)*(RL_ss - PSI)));
# Bcl_ss        = (I_ss*Rm_ss)/Yl - tr_ss;
# Bcs_ss        = mp_ss - (Bcl_ss/(RL_ss - PSI));
# Btl_ss        = Bl_ss + Bcl_ss;
# Bts_ss        = Bs_ss + Bcs_ss;




//=========================================================================
```

```
// LOG LINEARISED MODEL
// ======================================================================

// The full model is represented here.

//=======================================================================
// HOUSEHOLD
// ======================================================================

// 1. Euler Equation
(h/(Rd_ss*((c_ss - h*c_ss)^(sigmac))))*((sigmac/(1-h))*(c_hat - c_hat(-1)) + Rd_hat) + (chii)*Dl_hat =
        (h*betta_h/(PI_ss*((c_ss - h*c_ss)^(sigmac))))*((sigmac/(1-h))*(c_hat(+1) - c_hat) + PI_hat(+1));

// 2 + 3. Wage setting
f_hat = (1-betta_h*thetaw*(PI_ss)^((tauw-1)*(1-eta)))*((1-eta)*wstar_hat + lambdah_hat + eta*w_hat + N_hat) +
        betta_h*thetaw*(PI_ss)^((tauw-1)*(eta-1))*
        (f_hat(+1)-(1-eta)*(PI_hat(+1) - tauw*PI_hat + wstar_hat(+1) - wstar_hat));

f_hat = (1-betta_h*thetaw*(PI_ss)^(eta*(1-tauw)*(1+sigman)))*((eta*(1+sigman))*(w_hat - wstar_hat) + (1+sigman)*N_hat) +
        betta_h*thetaw*((PI_ss)^(eta*(1-tauw)*(1+sigman)))*
        (f_hat(+1) + eta*(1+sigman)*(PI_hat(+1) - tauw*PI_hat + wstar_hat(+1) - wstar_hat));

// 4. Real wage index
((thetaw*(PI_ss)^((tauw-1)*(1-eta)))/(1-thetaw)*((PIstarw_ss)^(1-eta)))*(PI_hat - tauw*PI_hat(-1) + w_hat - w_hat(-1))
      = wstar_hat - w_hat;

// ======================================================================
// FIRM
// ======================================================================

// 5. FOC Labour
w_hat(+1) = alppha*(K_hat(+1)) - alppha*(N_hat(+1));

// 6. FOC Capital
lambdaf_hat*lambdaf_ss - betta_f*(1-varphi)*lambdaf_ss*lambdaf_hat(+1) =
        alppha*(K_ss^(alppha-1))*(N_ss^(1-alppha))*((alppha-1)*K_hat + (1-alppha)*N_hat);

// 7. Marginal cost
mc_hat = (1-alppha)*w_hat + (1/(1-betta_f*(1-varphi)))*(lambdaf_hat - betta_f*(1-varphi)*lambdaf_hat(+1));

// 8. FOC Loans from merchant bank
(lambdaf_ss/Rc_ss)*(lambdaf_hat - Rc_hat) - ((theta*lambdaf_ss)/(Rc_ss))*(Lb_hat - Lb_hat(-1)) +
        ((betta_f*theta*lambdaf_ss)/(Rc_ss))*(Lb_hat(+1) - Lb_hat) =
        ((betta_f*psi_ss)/(PI_ss))*(psi_hat(+1) - PI_hat(+1)) +
        (betta_f^2)*(wpsi/2)*((1-psi_ss)^2)*Lb_ss*(-2*(psi_ss/(1-psi_ss))*psi_hat(+1) + Lb_hat);

// 9. FOC Default
(Lb_ss/PI_ss)*(1+ Lb_hat(-1) - PI_hat) = dpsi +
        betta_f*(wpsi/2)*(1-psi_ss)*(Lb_ss^2)*(1 - (psi_ss/(1-psi_ss))*psi_hat + 2*Lb_hat(-1));

// 10. Price setting
PI_hat = (betta_f/(1+betta_f*gammap))*PI_hat(+1) + (gammap/(1+betta_f*gammap))*PI_hat(-1) +
       (1/(1+betta_f*gammap))*((1-epsilon)/varrho)*mc_hat;
```

```
// ======================================================================
// DEPOSIT BANK
// ======================================================================

// 11. FOC Deposits
(1/Rd_ss)*(-Rd_hat - (pil_ss/(pil_ss+1))*sigmal*pil_hat) =
        betta_l*((1)/PI_ss)*(-(pil_ss/(pil_ss+1))*sigmal*pil_hat(+1) - PI_hat(+1))
        - (1/Rl_ss)*(Rl_hat + (pil_ss/(pil_ss+1))*sigmal*pil_hat)
        + ((betta_l*deltta_ss)/PI_ss)*(deltta_hat(+1) - PI_hat(+1) - (pil_ss/(pil_ss+1))*sigmal*pil_hat(+1));

// ======================================================================
// MERCHANT BANK
// ======================================================================

// 12. FOC Interbank loans
((1/Rl_ss))*((-1/(Rl_ss*((1/Rl_ss))))*Rl_hat - (pib_ss/(pib_ss+1))*sigmab*pib_hat) =
          ((betta_b*(deltta_ss))/PI_ss)*(((deltta_ss)/(deltta_ss))*deltta_hat(+1) - PI_hat(+1)
           - (pib_ss/(pib_ss+1))*sigmab*pib_hat(+1)) +
          ((betta_b)^2)*(wdelta/2)*((1-deltta_ss)^2)*Ll_ss*(-2*(deltta_ss/(1-deltta_ss))*deltta_hat(+1) + Ll_hat
          - (pib_ss/(pib_ss+1))*sigmab*pib_hat(+2))
          +(pib_ss/(pib_ss+1))*sigmab*pib_hat - ((pib_ss/(pib_ss+1)/PI_ss)*sigmab*pib_hat(+1) + PI_hat(+1));

// 13. FOC Loans to firms
(1/(Rc_ss))*(-Rc_hat - (pib_ss/(pib_ss+1))*sigmab*pib_hat) =
        betta_b*(psi_ss/(PI_ss))*(psi_hat(+1) - PI_hat(+1) - (pib_ss/(pib_ss+1))*sigmab*pib_hat(+1))
        +(pib_ss/(pib_ss+1))*sigmab*pib_hat - ((pib_ss/(pib_ss+1)/PI_ss)*sigmab*pib_hat(+1) + PI_hat(+1));

// 14. FOC Newly Issued Reserves
-(((pib_ss + 1)^(-sigmab))*Rm_ss)*(Rm_hat - (pib_ss/(pib_ss+1))*sigmab*pib_hat) +
        ((pib_ss + 1)^(-sigmab))*(pib_ss/(pib_ss+1))*sigmab*pib_hat = Rm_ss*eeta_ss*(Rm_hat + eeta_hat);

// 15. FOC Short Term Bonds
((pib_ss + 1)^(-sigmab))*(1/Rb_ss)*(-Rb_hat - (pib_ss/(pib_ss+1))*sigmab*pib_hat) =
           betta_b*((pib_ss + 1)^(-sigmab))*(1/PI_ss)*(-PI_hat(+1) - (pib_ss/(pib_ss+1))*sigmab*pib_hat(+1)) +
           betta_b*(kappas*eeta_ss/PI_ss)*(-PI_hat(+1) + eeta_hat(+1)) +
           (pib_ss/(pib_ss+1))*sigmab*pib_hat - ((pib_ss/(pib_ss+1)/PI_ss)*sigmab*pib_hat(+1) + PI_hat(+1));

// 16. FOC Long Term Bonds
((pib_ss + 1)^(-sigmab))*(1/(RL_ss-PSI))*(-(RL_ss/(RL_ss - PSI))*RL_hat - (pib_ss/(pib_ss+1))*sigmab*pib_hat) =
        betta_b*((pib_ss + 1)^(-sigmab))*(1/PI_ss)*(-PI_hat(+1) - (pib_ss/(pib_ss+1))*sigmab*pib_hat(+1)) +
        betta_b*(kappal*eeta_ss*RL_ss/((RL_ss - PSI)*PI_ss))*(-PI_hat(+1) +  eeta_hat(+1)
        - (PSI/(RL_ss - PSI))*RL_hat(+1))
        -betta_b*(RL_ss/((RL_ss - PSI)*PI_ss*(pib_ss + 1)^(sigmab)))*(-PI_hat(+1) -
        (pib_ss/(pib_ss+1))*sigmab*pib_hat - (PSI/(RL_ss - PSI))*RL_hat(+1))
        +(RL_ss/((RL_ss - PSI)*(pib_ss + 1)^(sigmab)))*(- (pib_ss/(pib_ss+1))*sigmab*pib_hat
        - (PSI/(RL_ss - PSI))*RL_hat(+1));

// 17. FOC Default
(Ll_ss/(PI_ss*(pib_ss+1)^(sigmab)))*(1 + Ll_hat(-1) - PI_hat - (pib_ss/(pib_ss+1))*sigmab*pib_hat)
        = ddelta + ((wdelta/2)*betta_b*(1-deltta_ss)*(Ll_ss^2)*
        ((pib_ss+1)^(-sigmab)))*(1-(deltta_ss/(1-deltta_ss))*deltta_hat + 2*Ll_hat(-1)
        - (pib_ss/(pib_ss+1))*sigmab*pib_hat(+1));
```

```
// ========================================================================
// CENTRAL BANK
// ========================================================================

// 18. Budget constraint
tr_ss*(tr_hat) - (Bcs_ss/PI_ss)*(Bcs_hat(-1) - PI_hat) + (Bcs_ss/Rb_ss)*(Bcs_hat - Rb_hat) =
        (RL_ss*Bcl_ss/((RL_ss - PSI)*PI_ss))*(Bcl_hat(-1) - (PSI/(RL_ss - PSI))*RL_hat - PI_hat) -
        Bcl_ss/(RL_ss - PSI)*(Bcl_hat - (RL_ss/(RL_ss - PSI))*RL_hat) + mp_ss*Rm_ss*(mp_hat + Rm_hat) -
        ((mp_ss*Rm_ss)/PI_ss)*(mp_hat(-1) + Rm_hat - PI_hat);

// 19. Feedback rule
Rm_hat  = rhoR*Rm_hat(-1) + (1-rhoR)*(rhoPI*PI_hat + rhoy*y_hat + rhody*(y_hat - y_hat(-1))) + em*std_m;

// 20. Eligible assets
I_ss*I_hat  = (kappas*Bs_ss/((PI_ss)*Rm_ss))*(Bs_hat(-1) - PI_hat - Rm_hat) +
        (kappal*Bl_ss*RL_ss/((RL_ss - PSI)*PI_ss*Rm_ss))*(Bl_hat(-1) - (PSI/(RL_ss - PSI))*RL_hat - PI_hat - Rm_hat);

// 21. Newly issued reserves
I_ss*I_hat = mp_ss*mp_hat - (mp_ss/PI_ss)*(mp_hat(-1) - PI_hat) +  emp*std_mp;

// 22. Balanced budget condition
mp_ss*mp_hat = Bcs_ss*Bcs_hat + (Bcl_ss/(RL_ss - PSI))*(Bcl_hat - (RL_ss/(RL_ss - PSI))*RL_hat);


// ========================================================================
// MARKET CLEARING + MISC
// ========================================================================

// 23. Market clearing
y_ss*y_hat = c_ss*c_hat + g_ss*g_hat + pif_ss*pif_hat + pil_ss*pil_hat
        + pib_ss*pib_hat + K_ss*K_hat - (1-varphi)*(K_ss)*K_hat(-1) +
        (wdelta)*((1-deltta_ss)^2)*(Ll_ss^2)*(-2*(deltta_ss/(1-deltta_ss))*deltta_hat(-1) + 2*Ll_hat(-2)) +
        (wpsi/2)*((1-psi_ss)^2)*(Lb_ss^2)*(-2*(psi_ss/(1-psi_ss))*psi_hat(-1) + 2*Lb_hat(-2));

// 24. Law of motion for K_hat
K_ss*K_hat = K_ss*(1-varphi)*K_hat(-1) + (Lb_ss/Rc_ss)*(Lb_hat - Rc_hat);

// 25. Indexing rule
w_hat(+1) - w_hat = (tauw*PI_hat);

// 26. Household BC
(Dl_ss/Rd_ss)*(Dl_hat - Rd_hat) + c_ss*(c_hat) - t_ss*t_hat
        = (w_ss*N_ss)*(w_hat + N_hat) + (Dl_ss/PI_ss)*(Dl_hat(-1) - PI_hat) - tr_ss*tr_hat;

// 27. Firm profit
pif_ss*(pif_hat) = y_ss*(y_hat) - (w_ss*N_ss)*(w_hat + N_hat) - ((psi_ss*Lb_ss)/PI_ss)*(psi_hat + Lb_hat(-1) - PI_hat) -
        (wpsi)*((1-psi_ss)^2)*(Lb_ss^2)*(-2*(psi_ss/(1-psi_ss))*psi_hat(-1) + 2*Lb_hat(-2));

// 28. Deposit bank profit
pil_ss*(pil_hat) = (Dl_ss/Rd_ss)*(Dl_hat - Rd_hat) - (Dl_ss/PI_ss)*(Dl_hat(-1) - PI_hat) +
        ((deltta_ss*Ll_ss)/PI_ss)*(deltta_hat + Ll_hat(-1) - PI_hat) -
        (Ll_ss/Rl_ss)*(Ll_hat - Rl_hat);
```

```
// 29. Merchant bank profit
pib_ss*(pib_hat) = - (I_ss*Rm_ss)*(I_hat - Rm_hat)
        + (Ll_ss/Rl_ss)*(Ll_hat - Rl_hat) - ((deltta_ss*Ll_ss)/PI_ss)*(deltta_hat + Ll_hat(-1) - PI_hat) +
        ((psi_ss*Lb_ss)/PI_ss)*(psi_hat + Lb_hat(-1) - PI_hat) - (Lb_ss/Rc_ss)*(Lb_hat - Rc_hat)  -
        (wdelta)*((1-deltta_ss)^2)*(Ll_ss^2)*(-2*(deltta_ss/(1-deltta_ss))*deltta_hat(-1) + 2*Ll_hat(-2))  +
         + (Bs_ss/PI_ss)*(Bs_hat(-1) - PI_hat) - (Bs_ss/Rb_ss)*(Bs_hat - Rb_hat) +
        (RL_ss*Bl_ss/((RL_ss - PSI)*PI_ss))*(Bl_hat(-1) - PI_hat - (PSI/(RL_ss - PSI))*Rl_hat) -
        (Bl_ss/(RL_ss - PSI))*(Bl_hat - (RL_ss/(RL_ss - PSI))*RL_hat);

// 30. Production function
y_hat = alppha*(K_hat) + (1-alppha)*(N_hat);

// 31 + 32. Balanced budget conditions
Dl_hat = Ll_hat;
Ll_ss*Ll_hat = mp_ss*mp_hat + (RL_ss*Bl_ss/(RL_ss - PSI)*PI_ss)*(Bl_hat - PI_hat(+1)
        - (PSI/(RL_ss - PSI))*Rl_hat(+1)) + Bs_ss*Bs_hat + Lb_ss*Lb_hat;

// ==========================================================================
// GOVERNMENT
// ==========================================================================

// 33. Gov Budget
g_ss*g_hat + (Bts_ss/PI_ss)*(Bts_hat(-1) - PI_hat) +
    (RL_ss*Btl_ss/(RL_ss - PSI)*PI_ss)*(Btl_hat(-1) - PI_hat - (PSI/(RL_ss - PSI))*Rl_hat)
    = (Bts_ss/Rb_ss)*(Bts_hat - Rb_hat)  + (Btl_ss/(RL_ss - PSI))*(Btl_hat - (RL_ss/(RL_ss - PSI))*RL_hat) + t_ss*t_hat ;

// 34. This is a budget balancing condition, need to include it.
Bts_ss*(Bts_hat) = Bcs_ss*(Bcs_hat) + Bs_ss*(Bs_hat);

// 35. Long run balanced budget condition
Btl_ss*(Btl_hat) = Bcl_ss*(Bcl_hat) + Bl_ss*(Bl_hat);

// 36. Growth rate of short run bonds
Bts_hat = gaama*(Bts_hat(-1) - PI_hat);

// 37. Growth rate of long-term bonds
Btl_hat = (RL_ss/(RL_ss - PSI))*RL_hat + rhob*(Btl_hat(-1) - (RL_ss/(RL_ss - PSI))*RL_hat(-1) - PI_hat) + elb*std_lb;

// 38 + 39. Shock to haircut
//kappas = rho_kap*kappas(-1) + ekp*std_kp;
//kappal*kappal_hat = rho_kapl*kappal*kappal_hat(-1) + ekp*std_kp;
end;

// ==========================================================================
// STEADY STATE CHECK
// ==========================================================================

resid(1);
steady;
check(qz_zero_threshold=1e-10);
```

```
// ========================================================================
// SHOCKS
// ========================================================================

shocks;
var em;
stderr 1;
end;

stoch_simul(irf=40);

// ========================================================================
// MODEL DIAGNOSTICS
// ========================================================================
// model_diagnostics(M_,options_,oo_)
```